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The Fine-Structure Constant from the Recognition Ledger

Light's stiffness at the smallest cell, carried to rest

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Abstract

With the matching between the ledger's lattice and the continuum derived at second order, under the record's statements and readings the paper names, and computed, the recognition ledger gives the fine-structure constant as one of two values, $1/\alpha = 137.0246 \pm 0.0072$ or 137.0193 ± 0.0073, below the measured value by 1.6 and 2.3 errors. One statement the record does not make decides between them: whether a stretch's own term in the count of the links' own quanta, moved along that stretch's proper time, stops where the stretch reaches the length of its history's line, or follows the stretch into the line. Colour lands at -0.40 and -0.57 errors and the weak share at -0.18 and -0.63, and the three couplings' chi-square with their correlations is 4.70 and 11.94. If the 2026 lattice determination's perturbative piece is as our recomputation finds, the result is 137.0341 and 137.0289, 0.26 and 0.98 errors from the measured value; the paper keeps the determination's printed value.

The fine-structure constant $\alpha \approx 1/137$ sets how strongly light and charge act on each other. It is measured to one part in ten billion; no accepted theory computes it. Here it is computed from the recognition ledger under the readings listed in section 8, with no continuous number fitted to light; these discrete readings were named with the measured couplings in view.

At the ledger's cell, $2\sqrt{\ln 2}$ Planck lengths on an edge, light's turn stiffness $1/e^2 = 1/(4\pi\alpha)$ is a whole count, 8, times the stiffness a long wave reads from the ledger's own carrier, $K = 1.040396 \pm 0.000002$. It is computed twice, independently, on the lattice of events with no measurement in it. Carried to rest by the Standard Model's computed running, $8K$ gives

$$\alpha^{-1} = 137.0246 \pm 0.0072 \quad \text{or} \quad 137.0193 \pm 0.0073,$$

against the measured 137.035999177(21): 84 and 122 parts per million below, 1.58 and 2.30 of their errors. The matching's second order is derived from the record's statements with one working reading, that an exchanged link meets a history as the link that is read does, and, where the links' own quanta meet one another, a count the record does not state, besides the first order's readings; it is computed from published two-loop weights, and for the top quark with the scalar from a weight derived from the action and checked against the published running, with no further choice. What the record does not fix in how the count of the links' own quanta treats a stretch at its line's length gives the two values; a second open choice in that count, which of two equal stretches is the line, is carried in the error. Version 1.7, which computed one part of it, gave 137.0234 ± 0.0107,

1.17 errors below the measured value. Colour lands with the same dressing (−0.40 and −0.57 standard errors) but admits every read of the carrier, and rests on an open premise of its own, that colour's links sit near no turn at the cell; the weak share (−0.18 and −0.63), which hardly depends on the dressing, lands with a form located with the measured share in view, so its landing is not an independent prediction, and two other located forms land it too, one built from the carrier alone (−0.36 and −0.82); the quarks' factor of three (−0.63) rests on fitted terms.

Light's read follows from what a coupling is, and the matching has no free constant if unrecorded histories are summed over every whole count: none at first order, and at second order the one the count gives. The counts follow from what an entry is, one comparison of its two sides priced once: other pricings miss, two events by thousands of errors, and the measured couplings over K, 8.0009 and 4.0019 under the first statement, 8.0013 and 4.0027 under the second, confirm it. It was first found with them in view. The other premises that carry the physics are readings identified with the couplings in view. The weight of the agreement is the coarse structure: light and colour at whole counts times one stiffness near 1.04 at a cell fixed by gravity, which a broad null gives once in 20 to 1,200; colour's place in that structure rests on that premise. Counting every form tried for where the logarithms end, the precision adds little: some candidate fits all three couplings as well with chance 0.23 and 0.35; on the menus fixed before the count the chance is 0.025 and 0.055, falling to 0.00003 and 0.00008 as the derivations found afterward are granted. Three forms tried fit better under the first statement and four under the second, all of them walks on the bonds, which the record excludes.

On the hadronic input $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$, which carries most of the error, the closure lands within 1.6 errors on four of the five lattice-based determinations under the first statement (one above its ask) and 2.4 below the fifth, and within 1.6 errors on three under the second (one on its ask) and 2.3 and 2.8 below the other two; it is strained or fails on every electron–positron evaluation with an error of 0.0001 or less, by 2.9 to 5.3 and 3.3 to 6.0 errors. The value it asks is filed as $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027882 \pm 0.000019$ (version 1.0 filed 0.027878 ± 0.000056); no determination made public since has scored it. With the second order computed it asks 0.027905 ± 0.000020 and 0.027943 ± 0.000020 under the two statements (version 1.8 filed 0.027939 ± 0.000048); on them the muon's leading hadronic part sits 2.3 and 2.8 errors high (a strain under both statements); on the perturbative piece as we recompute it, 1.3 to 1.4 and 1.8 to 1.9, inside the rule. On the earlier filing it sat 2.0.

Four further results sharpen the open structure without moving either headline value. First, counting a two-cycle closed history of one field's quanta once leaves the links a mass under the textbook gauge weight in every lawful gauge, and with the meeting rule taken whole removes the mass only while running at $-\frac{76}{3} = \frac{19}{17}(-\frac{68}{3})$ (where $-\frac{68}{3}$ is the weight's unit for $\frac{34}{3}C_A^2$ at $C_A = 2$). Within the resulting longest-stretch foliation, continuum proper-time integrands admit a one-parameter zero-mass family $\gamma \in [1, \infty]$ whose every finite member introduces two artificial boundary jumps; only the second statement ($\gamma = \infty$, $1/\alpha = 137.0193 \pm 0.0073$) eliminates both (86.9 per cent of its gap from 137.0246 coming from the first statement's far-end cusp weight at line-to-loop ratio $b = 1$), and finite discrete line-telescoping schemes cluster around it (+0.338 to +0.433 e-folds), while an exact foliation non-invariance theorem shows that a native discrete closure-preserving two-cycle kernel on the 24-bond lattice is required to fix the finite constant unconditionally. Second, one unchanged uniform-cut shared-link reader now derives all five two-loop running constants natively — pure-link $A = \frac{34}{3}$ (all seven face finite parts established analytically by ray integration, every $\ln 3$ cancelling, and total A invariant under cut rescaling), hopping-pair $C_s = 4, B_s = \frac{2}{3}$, and Weyl-fermion matter-hand $C_b = 2$, $B = \frac{10}{3}$ — closing the slope ratios $\frac{102}{121}$ and $\frac{26}{49}$ under four stated working limits. Third, a global SU(3) involution-manifold and Steiner-deltoid classification proves an exact no-go boundary at fixed curvature $S = 4$ — ruling out single-triangle pair costs, carrier dressing, five-mode neutrality, and pure-axis or pure-diagonal

even loops (including pure-axis eight-tick neutral cycles) — and bounds the quark sea well below the threshold (96 to 270 times short when speed-matched, 55.4 times at raw unit speed), leaving interior exact-$S = 4$ single-triangle costs (quartic coefficient $a < 9$) and a non-local 528-loop triangle-commutator pure-pair exit ($\eta_c = 0.00866$ harmonic, 0.00949 Euclidean Gaussian at $L = 4$; finite-coupling Monte Carlo open). Fourth, an independent four-loop three-coupling check at first-order cell matching on the 2024 inputs shows that the complete four-loop Standard Model gauge slopes move the rows by at most 0.044 of their errors, reconciles the pure $\overline{\text{MS}}$ and hybrid low-end electroweak determinations (97 per cent of their gap comes from the Fermi-constant relation alone, while the 2025 pure $\overline{\text{MS}}$ fit `SMDR` v1.3 lands colour, weak and hypercharge within one error at -0.40, -0.81 and -0.35), and confirms that every dispersive e^+e^- hadronic evaluation in the 2024 review is strained outside two errors on the hypercharge row.

Version 2.1, 26 November 2026. What changed is noted at the end.

1 The number

In 1916 Arnold Sommerfeld worked out how relativity splits the spectral lines of hydrogen into close pairs, the fine structure [1]. The size of the split was set by one pure number, which he called the fine-structure constant, $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$. It has no units. A civilization that counted in other units would find the same value.

Dirac's equation of 1928 explained the fine structure exactly [2], and it explained it in terms of α; it did not explain α. Arthur Eddington spent the rest of his life trying to: in 1929 he argued that $1/\alpha$ is exactly 136, and later that it is exactly 137 [3, 4]. Wolfgang Pauli worried at the number for decades; he died in 1958 in room 137 of the Red Cross hospital in Zurich [5]. In 1947 Lamb and Retherford measured a shift in hydrogen's levels [6], and Kusch and Foley an excess in the electron's magnetic moment [7], that Dirac's theory did not contain. Schwinger, Feynman and Tomonaga computed both from quantum electrodynamics, starting with Schwinger's $\alpha/2\pi$ [8]. Every one of those computations takes α as given. Feynman, who did as much as anyone to make them, wrote in 1985: "It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man" [9].

Measurement went on without explanation. The quantum Hall effect gave α from a resistance [10]. A single electron held in a trap gives it from the electron's magnetic moment, now to 1.1×10^{-10} [11]. Atoms recoiling from light give it from the ratio of Planck's constant to an atom's mass [12, 13]. The value recommended in 2022 is

$$\alpha^{-1} = 137.035999177(21) \quad [14].$$

At the energies of the large colliders the coupling was seen to depend on the length at which it is read: about $1/128$ at the mass of the Z boson [15]. That dependence is computed. The value at rest is not. In the Standard Model it is an input.

This paper computes it under the readings listed in section 8. The result is one of two values, $1/\alpha = 137.0246 \pm 0.0072$ or 137.0193 ± 0.0073, and the measured value lies above both, by 1.6 and 2.3 of their errors. The result depends on the hadronic input $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$, which carries most of the error and is taken from the 2026 lattice determination: the closure holds within two errors on most lattice determinations at the physical pion mass, and it is strained or fails on every electron–positron evaluation with an error of 0.0001 or less (section 6). The carry uses the Standard Model's fields alone between the Z and the cell: one more charged fermion

at 1 TeV would raise each value by 7.764, about 1,070 of its errors, so the closure says there is no such field (section 3). The matching between the lattice and the continuum is derived at second order from the record's statements and readings the chain's table marks (section 8): an exchanged link meets a history as the link that is read does, and for the top quark with the scalar the scalar's mass is held, like the bodies', at the pole of its dressed line; where the links' own quanta meet one another, the longest stretch is the line, a count the record does not state but which meets the two conditions it fixes; and, as at first order, the links' own turns are not summed as weights at the cell. It is computed with no further choice, from published two-loop weights and, for the top quark with the scalar, a weight derived from the action and checked against the published running: the histories in which the link that is read meets the exchanged link's own line, the scalar rings, those in which the links' own quanta meet one another, and the top quark's closed history with the scalar inside. The count of the links' own quanta leaves two choices the record does not make. The first gives the two values: where a stretch of a history grows to the length of the history's line, its own term either stops there and is counted once, or follows the stretch into the line; section 2.5 states both exactly. The second, which of two equal stretches is the line, is carried in the error. One statement on the record would decide between the two values; until then the paper gives both, and wherever a number depends on it, both are given, the first statement's first. The top quark with the scalar, the last part, is computed too; it moves each value up in its last printed digit. Version 1.7, which computed one part, gave 137.0234 ± 0.0107. No continuous number is fitted to light, and the steps that are readings were named with the measured couplings in view; section 8 marks each.

Section 2 states what the recognition ledger gives for light at its smallest cell, which of its steps follow from stated premises, and which are readings. Section 3 carries the measured couplings from rest to that cell. Section 4 compares the two, states the fine-structure constant the ledger gives, and counts what the agreement weighs. Section 5 does the same for colour, for the weak share and for the quark masses, and gives two numbers the package was never compared with before they were filed. Section 6 says where it does not close. Section 7 files the numbers that test it. Section 8 lists every step of the chain with its status.

2 Light at the ledger's cell

2.1 The ledger, its bonds and its cell

The recognition ledger records every event as a posting on one of four accounts, each with a debit and a credit column [18]. Counted on both columns, the events form a lattice in four dimensions, the integer points together with the integer points shifted by $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. Its twenty-four shortest vectors, the bonds, are the sixteen half-integer vectors $(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$, which are the eight debits and the eight credits, and the eight unit vectors $\pm e_i$, which are a debit with its credit on one account, its reverse, and the six edges of the record cube [18]. All twenty-four have one length, and the lattice's symmetries carry any bond to any other. Every event has twenty-four bonds and every bond joins two events, so there are twelve bonds for every event. Every bond lies in eight triangles of bonds, and a triangle has two of its bonds at each of its corners, so ninety-six triangles have a corner at each event; every triangle has three corners, so there are thirty-two triangles for every event. Every triangle closes one unit bond

with two half bonds, one a debit and the other a credit, so that each loop is one recognition closed by a unit step; neither kind of bond makes a triangle by itself. In four dimensions a point has at most twenty-four nearest neighbours when no two points are closer to each other than it is to its nearest neighbours [21]. A lattice meets this condition at every point, since the difference of two of its points is again a lattice vector, so twelve bonds per event is a count of this lattice and not a setting; the smallest change a lattice allows, one bond more or fewer per event, would move light at the cell by forty-two of its errors or more.

Light is the loop law on the triangles of those bonds: a link carries a turn, and light's coupling is how stiff that turn is. Colour is the same loop law on an account's three partners.

The lattice has a size. One bit of record per λ^2 of area gives Newton's constant as $G\hbar = \lambda^2 c^3/(4\ln 2)$, so the cell's edge is

$$\lambda = 2\sqrt{\ln 2}\,\ell_P = 1.665109\,\ell_P \quad [19],$$

and the energy at which light is read at the cell is $E_P/1.665109 = 7.3322\times10^{18}$ GeV, with $E_P = 1.22089\times10^{19}$ GeV the Planck energy. The edge is fixed on gravity's bit and is used here unchanged: it is taken from the measured Newton's constant. Under the count of section 2.5 the closed histories of the known fields themselves supply 1.57 per cent of gravity's stiffness, and the cell's one bit a face is read as counting the whole stiffness, these included; were it to count the record's own stiffness only, the cell would be 0.78 per cent larger and light's pull -2.23 and -2.94. Where, at that edge, a coupling is read is the matching's question (section 2.5).

2.2 The counts

At the cell a link's turn stiffness, before anything dresses it, is a whole count, and the count is set by the lattice once two things are said: how a loop is read, and on which body. On the twenty-four bonds, for every uniform field strength the squared loop values of the triangles (the field strength's flux through each) sum, per unit four-volume, to exactly $F_{\mu\nu}F^{\mu\nu}$; the triangles' second moment is the same in every direction. So if a loop costs $S\cdot\frac{1}{2}\theta^2$ for a small turn θ in each direction in which the hopping body turns, each loop is dressed by K, and the body's generator squares to trace T, then the lattice's stiffness equals the continuum coupling for every field exactly when

$$\frac{1}{g^2}\Big|_{\text{cell}} = 2\,S\,T\,K.$$

Colour is read on an account's three partners, where each generator has $T=\frac{1}{2}$. Light turns one of a body's two columns and leaves the other still, so its generator is a projector of rank one and $T=1$. The weak link and hypercharge, read on the same two columns, have $T=\frac{1}{2}$ each, and light's trace is their sum, $\operatorname{tr}Q^2 = \operatorname{tr}T_3^2 + \operatorname{tr}Y^2 = \frac{1}{2}+\frac{1}{2}$; that is the linear algebra of light's trace, not a statement that each part has $4K$ (section 5). The ratio of light's count to colour's is therefore the ratio of their generator traces, two, for every S: a unit charge against the fundamental's one half, as it is between the $U(1)$ and $SU(N)$ loop actions of field theory at equal cost per turn.

What sets S is how a comparison is costed. The reading adopted here costs it on its pair, the debit reading over the credit reading. For a comparison z whose credit reading is z^{-1} the cost

is that of the pair $z/z^{-1} = z^2$, one formula for every comparison. The cost of a comparison is never negative: on a size $x > 0$ it is $J(x) = (x + x^{-1})/2 - 1$, on a turn $e^{i\theta}$ it is $1 - \cos\theta$, and on a comparison z that carries both it is $|z-1|^2/(2|z|)$, which reduces to each of these. For a loop the credit reading is the loop's reverse, so the pair reads the turn twice,

$$1 - \cos 2\theta = 4\,(1 - \cos\theta) - 2\,(1 - \cos\theta)^2,$$

and $S = 4$:

$$\frac{1}{g_3^2}\Big|_{\text{cell}} = 4K, \qquad \frac{1}{e^2}\Big|_{\text{cell}} = 8K,$$

where K is the one dressing both read (section 2.3) and $1/e^2 = 1/(4\pi\alpha)$ in the usual notation. Read on one side only the counts would be 2 and 1, and with three senses 18 and 9: light would then miss by thousands of its errors and colour by hundreds. The pair reading was first found with the measured couplings in view (the measured couplings over K are 8.0009 and 4.0019 under the first statement, 8.0013 and 4.0027 under the second), and it is what an entry is: one comparison of its two sides, the debit read against the credit, priced once. Light and colour share it and both confirm it, and on a posting's golden entry it also gives the matching's unit its number (section 2.5). Colour's confirmation rests on an open premise of its own, that colour's links sit near no turn at the cell (section 2.4). On a bond of the carrier the pair reads the bond's ratio squared, in the carrier's weight and in light's read alike; that only relabels the carrier's variable by two, and it leaves K unchanged. The weight uses the cost. Light's read uses a second object, the mean of the debit and credit readings, $(z + z^{-1})/2$: a size alone raises it above one by the size's cost, a turn alone lowers it below one by the turn's cost, and on a bond $e^{u+i\varphi}$ that carries both, its real part is the product of the size's read and the turn's read, $\cosh u\,\cos\varphi$ (section 2.4). Taking the cost in place of the mean would leave light undressed, at 8 in place of $8K$: 564 and 563 errors low. On one golden entry the pair's cost is exactly one half (section 2.5).

The reading can be said in the record's own terms. A hop's cost compares what the body reads with what the rest of the record reads. The record is neutral: the turns of an account's five modes multiply to one, so a body and the rest of the record read inverse turns, and their comparison is the body's turn squared, which is the pair. With nothing on the credit side the same comparison reads the body's own turn, which is the one-sided reading. With k of a body's three partners on the credit side the number of senses is $((3 + k)/3)^2$: 1, 16/9, 25/9 or 4. Four is the most a neutral record allows, and light asks exactly that (4.0004 ± 0.0003 and 4.0006 ± 0.0003; with two partners light would be 4,429 and 4,413 errors low). Among costs that depend on two kept records only through their comparison, the pair is the one calibrated at balance; the cost of the pair's square root, z itself, is just as free of a common change of reference and is the one-sided reading. The credit side is the whole rest of the record because nothing less balances the entry: a part of the record balances a body on every neutral link exactly when it is the rest of the record, and of the counts above only four is a double entry. The entry is priced once because it is one thing: its debit and its credit are its two sides, not two things joined, and asking whether they are one event or two adds a split the entry does not have. Priced as two events, the sum applied inside one entry, light would be 7,247 and 7,219 errors low; priced as nothing, light would have no stiffness at all. The record's own sums, the cost of a ledger, of its first tick and of a double entry at a strain, add postings, and that posting-level sum is the matching's unit (section 2.5). If the cost of a record obeys the

record's law of composition, a premise, an entry's two sides held in one record cost either nothing, taken together at balance, or the cost of z^2, one taken against the other, and never the sum of the costs of z and $1/z$; so a link that is stiff at all has the count four. Light and colour measure it, and the end of the logarithms takes the same price (section 2.5); reading one side, each side against balance, or the product misses by thousands of errors.

The costs that give these counts do not see every part of a link. Light's pair does not see a half turn of light's link: $1-\cos 2\theta$ vanishes at $\theta=\pi$. Colour's cost does not see the overall turn of the partners' link at all, so turning that link by any sixth root of one costs nothing: light's link, its determinant, turns by at most a half turn. If the ledger summed over these unseen turns, a history would survive only if, on every link, the unseen turns picked up by all the bodies crossing it cancelled, and neither a free electron nor a free quark would exist. So a record holds each link's unseen part. That is a second reading; the quarks' thirds need it whatever form the count takes.

Carried to the cell (section 3), the measured couplings give

$$\frac{1}{e^2}-\frac{2}{g_3^2}=-0.0030\pm 0.0098 \quad \text{and} \quad -0.0044\pm 0.0098,$$

which is colour's landing said again, not an independent check; what it adds is that the factor two, the trace, holds at the cell fixed on gravity with no dressing in it.

2.3 The dressing

The carrier is one real number on every event, the logarithm θ of its ratio. A bond b between two events carries the difference $\ell_b=\theta_x-\theta_y$, and the cost of a bond is the ledger's cost of a comparison, $J(e^{\ell})=\cosh\ell-1$ [20]. The carrier sits at weight

$$P[\theta]\propto\exp\Big(-\sum_b\big(\cosh\ell_b-1\big)\Big)$$

with coefficient one (section 2.4). Because the weight depends only on differences across bonds, moving every θ_x by the same amount changes nothing; the hybrid Monte Carlo runs below hold that uniform shift at zero, $\sum_x\theta_x=0$, because the starting field, every momentum the sampler draws and every force it applies have no component along it. Rescaling every θ_x by one common factor within the $N-1$ directions that remain (pinning one event instead leaves the same number of directions) gives, with $\mathcal{S}=\sum_b(\cosh\ell_b-1)$ so that $P[\theta]\propto e^{-\mathcal{S}}$, the identity $\sum_x\langle\theta_x\,\partial\mathcal{S}/\partial\theta_x\rangle=N-1$ for a lattice of N events, which becomes, averaged over the $12N$ bonds,

$$\langle\ell\sinh\ell\rangle=\frac{N-1}{12N}\;\longrightarrow\;\frac{1}{12},$$

the twelve being the number of bonds per event. The average over the bonds is exact at every size; on the infinite lattice the bonds are all alike, so the limit holds for each. That fixes how much each bond fluctuates. If the fluctuation of a bond were exactly Gaussian with variance s, the identity would read $s\,e^{s/2}=1/12$, so $s=0.0800633$, and the mean loop weight would be $\langle\cosh\ell\rangle=e^{s/2}=1.040844$. Computed directly on the lattice it is

$$K_m=\langle\cosh\ell_b\rangle=1.0408495\pm 0.0000001,$$

a little above the Gaussian value because the bonds have thinner tails than a Gaussian: their fourth moment is 0.7 per cent below $3s^2$. That is the dressing a single loop at one place reads: the mean.

Light is a different field. It lives on the links, and its energy is $\sum_T w_T F_T^2$ over the triangles T of the bonds, where F_T is the field's value around the triangle and w_T is the mean of $\cosh \ell$ over the triangle's three links. Light's stiffness at long wavelengths is the energy per cell of a uniform field strength f, minimized over periodic link fields A:

$$K = \min_A \frac{1}{N} \sum_T w_T \big(f_T + (\operatorname{curl} A)_T\big)^2 \Big/ \frac{1}{N} \sum_T f_T^2 .$$

With every w_T equal to the mean, K would be the mean. Because the weights spread, the field relaxes into the softer triangles and K falls below the mean. On every configuration of the carrier the relaxation is bounded: the minimum lies between the harmonic and the arithmetic mean of the triangle weights. The arithmetic mean is the energy of the field at rest, which is one of the fields tried; the harmonic mean follows from the Cauchy–Schwarz inequality, because a uniform field strength has no divergence on this lattice (the eight triangles on a link carry fluxes that cancel). To second order in the spread,

$$K = \bar{w} - \frac{1}{\bar{w} \sum_T f_T^2} \sum_q \operatorname{tr}\big(\Pi(q)\, \langle g\, g^\dagger \rangle(q)\big), \qquad g_T = (w_T - \bar{w})\, f_T ,$$

with Π the projector onto the loop values a periodic field can make and $\langle g\, g^\dagger \rangle(q) = \sum_r e^{-iq\cdot r} \langle g(r) g(0)^\dagger \rangle$ the momentum-space covariance. With the covariance of the weights taken from a Gaussian carrier of the same bond variance ($s\, e^{s/2} = 1/12$), the gap on lattices of 4^4, 6^4 and 8^4 cells is 0.000466, 0.000472 and 0.000473, which gives $K = 1.040377$. With the carrier's exact covariance, the same expression evaluated on sampled configurations, the gap is 0.000466 and $K = 1.040384$.

At all orders the minimization is solved exactly. The carrier is sampled from $P[\theta]$ by hybrid Monte Carlo on periodic lattices of L^4 cells, $L = 4$ to 16. On each sampled configuration, for each of the six coordinate planes of the field strength, the periodic link field of least energy is found to a relative residual of 10^{-11}, and the results are carried to infinite volume in $1/L^4$. The gap is 0.000453, and

$$K = 1.040396 \pm 0.000002 .$$

An earlier computation gave 1.040399 ± 0.000005. The two were written independently and agree within their errors; the code, the runs and the analysis of the computation used here are public (see Code and data, at the end). On the samples the identity $\langle \ell \sinh \ell \rangle = (N-1)/(12N)$ holds at every size, and each of the 181,200 minimizations lies inside its two bounds. Of the difference between the Gaussian second order and all orders, 0.0000197, the exact covariance accounts for 0.0000070 and the orders beyond the second for 0.0000127.

2.4 Which stiffness light reads

The carrier has three stiffnesses: the mean loop weight $K_m = 1.0408495$, the long-wave value $K = 1.040396$, and its own stiffness against a slow twist of its own field, 1.040759 ± 0.000002 (an earlier computation gave 1.040747 ± 0.000007). Which one is light's follows from what a

coupling is. How a triangle weighs its three links, and whether light changes the carrier it reads, rest on two readings, stated as such below.

The read. A coupling is the coefficient of the squared field strength in a field's law at long waves. For light on the carrier that is the energy per cell of light's field at a uniform loop value f, with light's links obeying their own law in the carrier's weights. On one configuration of the carrier this energy splits exactly into two parts: the cost of f once the links have settled into the weights, and the cost of the field's departure from that settled shape, which does not depend on f. So the coefficient is the relaxed stiffness, the long-wave value, configuration by configuration. The mean loop weight is the cost of a uniform loop value whose links have not settled. It bounds K from above, and it is what one loop at one place reads, not what light's long wave reads.

Not the carrier's own stiffness. For a bond of size u carrying a turn φ, the comparison reads the complex ratio $z = e^{u+i\varphi}$, and two things built on that ratio must be kept apart. Its cost is never negative: $\cosh u - 1$ on a size and $1 - \cos\varphi$ on a turn (section 2.2). Its read is the mean of its two readings z and $1/z$, $(z + 1/z)/2 = \cosh(u + i\varphi)$, which a size raises to $\cosh u$ and a turn lowers to $\cos\varphi$; on a bond carrying both, the real part of the read is the product of the two, $\mathrm{Re}\cosh(u + i\varphi) = \cosh u\,\cos\varphi = \cosh u - \cosh u\,(1 - \cos\varphi)$, and the dressing is read from the read, not from the cost. The turn takes $\cosh u\,(1 - \cos\varphi)$ from the real part of the read: with stiffness $\cosh u$, even in the turn, and never multiplying $\sinh u$, so no current runs between light's turn and the carrier's size. The carrier's own stiffness is its response to a twist of a field that lives on events, and such a twist has loop value zero around every closed triangle. It is never light's.

The weighting, a reading. We read each link as reading the whole of a loop's turn with its own stiffness, so that a loop's cost is the sum of what its links read: the links act in parallel. Given that, a triangle's weight that is unchanged when the lattice's symmetry cycles its three links, and equal to k when every link has stiffness k, is the arithmetic mean of the three. The alternative reading shares the loop's turn among the links in series, each taking a part inversely to its stiffness; it gives the harmonic mean, and light then misses by 22 and 23 errors. One link's weight alone breaks the lattice's symmetry, the product of the three is not k when every link has stiffness k, and the geometric mean is not a sum of what the links read.

Light reads the carrier without weighing it, a reading. Light's law depends on the carrier through the triangle weights. If light's turns were summed as weights together with the carrier, the sum over light would leave on each configuration of the carrier a factor that depends on that configuration, and the carrier's law would shift: at first order its coefficient would become about $35/24$, K would be 1.027995, and colour, whose ask does not depend on the hadronic input, would miss by ten of its errors under the first statement and eleven under the second. We read the cell the other way: the links' own turns are not summed as weights there, the same reading that keeps light's own loops out of the matching (section 8). Nothing then multiplies the carrier's weight, and the coupling is the relaxed stiffness averaged over the carrier's own law. What motivates the reading is that light's turn term, a stiffness times $1 - \cos F$, is not a cost that obeys the ledger's law, while a real cost that does obey it gives every turn the weight of the size alone [18]. That motivates the reading; it does not prove it. The same reading does not, by itself, carry colour from the cell. The pair does not see a half turn of the partners' link either: for colour's loop, whose turn is a matrix U, the pair's cost

is $\mathrm{Re\,tr}(1-U^2)$, which vanishes on every turn whose square is one. If the links' own turns were summed as weights at the cell, each triangle weighing $e^{-\mathrm{cost}}$, the pair alone would not keep a triangle near no turn. On the 24 bonds the mean of $\frac{1}{3}\,\mathrm{Re\,tr}\,U$ over triangles comes out -0.19 at stiffness K a triangle, and -0.29 and -0.32 at twice and four times that: it runs to the half turn's $-\frac{1}{3}$, not to one. Four times the one-sided cost has the same curvature, so the same counts, and gives $+0.89$, $+0.94$ and $+0.97$. The counts cannot tell the two apart. Not summed as weights, the turns are still not small under the pair alone. The pair alone then leaves a triangle's mean reading anywhere from $-\frac{1}{3}$ to one at no cost, and on the 24 bonds the least state reads -0.27, -0.30 and -0.32 at K, twice and four times that; runs from every start tried end below zero. So colour's carry from the cell, which takes the links' turns as small, needs something that prices the half turn, and the pair does not. That carry therefore rests on a second open premise, which the record does not state: that colour's links sit near no turn at the cell. The read rests on the first, that the links' own turns are not summed as weights at the cell; a statement of it on the record would settle the read, not colour's carry.

Structural boundary and global classification of colour's ground. Write $\bar{q} = \langle \frac{1}{3}\,\mathrm{Re\,tr}\,U\rangle$ for the mean triangle reading. Why colour alone breaks away from no turn under the pair cost is an exact group-manifold theorem. On SU(N), the pair cost $V_{\mathrm{pair}}(U) = \mathrm{Re\,tr}(I_N - U^2) = \frac{1}{2}\|U - U^\dagger\|_F^2$ vanishes on the involution union $\bigsqcup_{0\le 2k\le N} \mathrm{Gr}(2k, \mathbb{C}^N)$, with flat dimension $d_k = 4k(N-2k)$ and stiff dimension $s_k = (N^2-1) - 4k(N-2k)$. Both light (U(1)) and the weak link (SU(2)) have $d_k = 0$ (discrete zero-dimensional center elements, all modes stiff); colour (SU(3)) is the unique smallest SU(N) (and sole Standard Model) gauge group with a positive-dimensional non-central involution manifold $H \cong \mathbb{CP}^2$ ($d_1 = 4$, $s_1 = 4 = s_0/2$), halving the 24-bond kept-law harmonic least level from 127.0156 to 63.5078 per site on box side $L = 3$ ($L = 8$ finite-grid values 127.0425 versus 63.5212). At fixed small-turn curvature $S = 4$ ($1/g_3^2 = 4K$), a sharp no-go boundary rules out every standard local and even-loop mechanism on the 24-bond lattice: (i) splitting the twelve bond directions into three frames of four (one axis frame and two diagonal frames, with bond counts n_0, n_1, n_2 along a closed loop of length ℓ_{loop}), the 24-bond three-frame parity theorem gives $n_0 \equiv n_1 \equiv n_2 \equiv \ell_{\mathrm{loop}} \pmod 2$ on every contractible loop (and $n_0 \equiv \ell_{\mathrm{loop}} \pmod 2$ on every periodic torus), so placing $U_{\mathrm{axis}} = h \in H$ and $U_{\mathrm{diag}} = I_3$ sets every even-length loop — four-bond squares, 60° rhombi, two-triangle composites, six-cycles and eight-tick neutral cycles — identically to I_3; in the exact 12×12 Bloch spectrum, pure-axis even loops (including pure-axis eight-tick neutral cycles) and pure-diagonal even loops never cross $E_0(H;\eta) > E_0(I;\eta)$ for any weight $\eta \ge 0$, while mixed-frame even loops cross only at unphysical $\eta_c \gg 1$ (nonzero-area shared-diagonal two-triangle composites at $\eta_c = 103.5$, shifting $4K$ by 1,656×; zero-area mixed six-bond and eight-bond loops at $\eta_c = 46.13$ and 14,937, with zero continuum shift to $4K$); (ii) carrier dressing ($w_T \ge 1$) preserves $E_0(H; w_T) = \frac{1}{2}E_0(I; w_T)$ configuration by configuration, five-mode neutrality ($\det h = +1$) admits (h, I_2), and SU(5) adjoint unification forces $1/e^2 : 1/g_3^2 = 8/3 \ne 2$; (iii) matter determinants are decisively insufficient: naive Dirac quarks ($r = 0$, arbitrary mass) obey exact unitary equivalence ($\Delta \equiv 0$), and $N_f = 6$ Wilson–Dirac quarks (whose equilateral-triangle spin trace $4r(r^2 - \frac{3}{2})$ changes sign at $r_* = \sqrt{3/2}$ for large m and 1.3533 at $m = 0$) favour I_3 at $r = 1, m = 0$ by only $+0.4073$/site in four-dimensional Euclidean action versus the 55 to 110/site gluon gap from four flat versus four stiff modes across K to $4K$ (135 to 270 times short) and by $+1.1473$/site in the raw unit-speed ($c_q = 1$) three-dimensional Dirac sea (55.4 times short of 63.508/site), or $+0.6624$/site (95.9 times short) when matched to the 24-bond

gauge-wave speed $c_{\text{gauge}} = 1/\sqrt{3}$. Over the entire SU(3) maximal torus ($\theta_1 + \theta_2 + \theta_3 = 0$), writing $u = 1 - \frac{1}{3}\operatorname{Re}\operatorname{tr} U \in [0, \frac{3}{2}]$ and $y = \operatorname{Im}\operatorname{tr} U$, the complete two-parameter exact-$S = 4$ single-triangle class-function family $V_{a,b}(u, y) = 12u - au^2 + by^2$ is globally nonnegative if and only if $a \leq 9$ (H boundary) and $a - 3b \leq 8$ ($\mathbb{Z}_3$ boundary), with no third boundary on the Steiner deltoid. The pair cost $V_{\text{pair}} = V_{9,1}$ sits on the degenerate $a = 9$ boundary (uniquely pinned by one-dimensional eigenvalue doubling), while interior exact-$S = 4$ single-triangle costs ($a < 9$) stabilize zero turn cleanly: for instance, $V_{\text{agg}} = V_{6,0} = 3(4u - 2u^2)$ gives $\bar{q} = +0.609, +0.846, +0.927$ in the one-triangle kept-law ground state (Euclidean Weyl integral $+0.179, +0.710, +0.910$) and $+0.861, +0.940, +0.971$ in 24-bond cold Monte Carlo at $K, 2K, 4K$, and $V_\kappa = V_{9-\kappa,1} = V_{\text{pair}} + \kappa u^2$ crosses at harmonic thresholds $\kappa_c^{\text{harm}} = 1.175, 0.560, 0.274$ (24-bond Monte Carlo cold-start crossings in $[1.0, 1.5], [0.50, 0.75], [0.25, 0.40]$ and full hysteresis brackets $[1.0, 2.5], [0.50, 1.50], [0.25, 0.80]$ across $K, 2K, 4K$). A non-local pure-pair exit also preserves $4K$ and $E_0(I)$ identically: triangle-commutator loops $W_{12} = U_{T_1} U_{T_2} U_{T_1}^\dagger U_{T_2}^\dagger$ satisfy $V_{\text{pair}}(W_{12}) = O(F^4)$ around I_3 and $8\operatorname{tr}((F_{2a} - F_{1a})^2)$ around H. While individual commutator families retain exact non-gauge flat zero modes on H, combining the 32 adjacent-triangle and 496 general-pair commutators (528 loops per site) lifts all local and hyperplane non-gauge zero modes at $L = 4$ (leaving only one local twisted-gauge mode per site plus one global $\mathbb{CP}^2$ mode, $z_a/\text{site} = 1 + 1/512$) and crosses in favour of I_3 at $\eta_c = 0.00866$ (kept-law harmonic) and $\eta_c = 0.00949$ (Euclidean Gaussian); whether that non-local pure-pair exit stabilizes $\bar{q} > \frac{1}{3}$ in finite-coupling 24-bond Monte Carlo remains open (code archive, `open_premises_analysis/`).

Computed alternatives miss light on the carry of section 3: one link's weight by 6.8 to 9.0 errors under the first statement and 7.5 to 9.7 under the second, the geometric mean by 11.8 and 12.5, the harmonic mean by 22 and 23, the product by about twelve hundred. The read and both readings were identified with the landing known. What the read adds is that it follows from what a coupling is; what each reading adds is a reason that does not mention a coupling, and an alternative that misses.

The coefficient. One identification is left under the read: each comparison weighs its history by e^{-J}, with J the comparison's own cost, so the cost acts as a weight at coefficient one. The cost's unit is not a setting: changing it relabels the carrier's variable, and the dressing does not see a relabelling. Of the two discrete alternatives the chain table of section 8 counts, the cost placed in the amplitude instead of the weight would put light nearly three hundred errors low and colour seventeen, and a coefficient of one half would put light more than five hundred errors high. Light locates the coefficient at 0.9972 ± 0.0018 and 0.9959 ± 0.0018; that is light's landing said for the coefficient, not a second confirmation of it. The matching's unit rests on the same identification, and through it the combination of light and colour in which the dressing cancels, whose error is mostly colour's, locates the coefficient at 0.974 ± 0.084 and 0.962 ± 0.083.

2.5 Matching the lattice to the continuum

A stiffness on a lattice and a coupling in the continuum are compared through a matching, which in general has a scale and a constant. If the ledger's lattice were a regulator, with the fields' own fluctuations summed along its bonds, the constant would be large: the links' own loops summed at the cell take 2.54 from light and 1.52 from colour, and matter summed along

the bonds adds 0.12 to light. Each moves light by hundreds to thousands of its errors, and the combination of light and colour that contains no dressing, measured at -0.0030 ± 0.0098 and -0.0044 ± 0.0098, would stand at $+0.50$ with the links' loops summed. Read as a regulator, the lattice is excluded by measurement.

The reading adopted here counts the histories that leave no record differently. Such a history has no count on the record, and its count is read as summed: every whole number of units from one, each weighing one over itself. If $q(t) = \sum_i w_i e^{-T_i t}$ is the return weight of one unit, with weights $w_i \geq 0$ that sum to one and proper times $T_i > 0$, at $t = k^2 + m^2$ for a wave number k and a rest m, the sum is $\sum_{n\geq 1} q^n/n = -\ln(1-q)$, and for $0 < t \leq T_1/(2M_2)$, with T_1 and M_2 the moments defined in the display,

$$0 \;\leq\; -\ln\big(1-q(t)\big) - \ln\frac{1}{T_1 t} \;\leq\; \frac{2M_2 t}{T_1}, \qquad T_1 = \sum_i w_i T_i, \quad M_2 = \sum_i w_i T_i^2.$$

The one over the count is not put in by hand: marking one recognition of a closed history opens it, so the n markings of a closed history of n recognitions are n open histories, and the closed history weighs one n-th of the open one. The same weight is the logarithm of the number of whole occupations of a mode. So a field adds its coefficient times one logarithm and nothing else, whatever the shape of one unit, and the same for every spin, because the count does not see how a history's length is divided between its stretches: the orbital and the spin parts of every field's coefficient are multiplied alike, where a cut on each stretch would weigh them differently. At first order the comparison has no constant, and a constant common to every field is the same thing as a different unit: the constant and the scale are one number, the proper time T_1 of one unit, and the logarithms end at $1/\sqrt{T_1}$. In the language of the field-theory literature this is the dimensional-reduction scheme with every threshold a pure logarithm; it differs from minimal subtraction by the constant $C_A/(48\pi^2)$ in each $1/g^2$, where C_A is 2 for the weak group and 3 for colour [22]. That is the matching at first order in the couplings; the result is compared with the second order included, which follows. At second order a link is exchanged inside a closed history, and the count must say how the stretches between the places where the link meets the history are weighed. The first order has already said it for the link that is read: the count sees the history's whole length and not how it is divided, which is what gives every spin the same end, and the three couplings fix that end to within 0.0076 of an e-fold. Counted instead in whole recognitions, the stretches would move the end of the spin-one-half loops by 0.60 of an e-fold (by 0.15 in half recognitions), whether or not a stretch of no length is allowed. We take an exchanged link to meet a closed history in the same way as the link that is read: one field, one way of meeting. This is a working reading, not a theorem. The statement it extends speaks of the link without saying which, and the first order that fixes it was settled before the second order was computed, so it is not chosen with light in view. Under it, the rules that count each cycle or each stretch of the history in whole units are excluded. It leaves one question open: how the exchanged link's own stretch is counted, within the history's one whole count, by a whole count of its own, or not at all.

Which stretch counts how follows from what a history is on the record. A history is one thing's: a body's history at rest is its own count of recognitions, and a history with no record inside it is summed over its count, every whole number from one, a line plainly and a loop at one over its count. A body's loop with a link exchanged inside it is therefore two histories, not

one. The body's loop keeps its own whole count, at one over the count, with the exchanged link's two ends anywhere along it, as the ends of the link that is read are. The exchanged link's stretch is a line of the link's own quantum, which the record counts like every body (its closed histories already enter the first order so); it has no record inside, so it is summed over its own count, plainly, from one. Two other counts are possible in form and are not taken. Counting the link's stretch into the body's one count would need one count over the recognitions of two things, and the record has no history of two things; giving the link's stretch no count of its own would leave a thing's line uncounted. Each also fails when computed: where the link is short and its two ends are close, each leaves a corner that no count reaches, and its logarithm diverges. The count taken here is the only one of the three that is complete. It uses nothing beyond the counting the first order already rests on, and one working reading: that an exchanged link meets a history as the link that is read does.

With this count the second order can be worked out wherever the exchanged link is another thing than the body and the link that is read meets the body only: the closed histories of the quarks and the leptons with a colour, a weak or a hypercharge link inside. The exchanged link's line, summed over its whole counts, is $\lambda^2/(e^{\lambda^2 k^2}-1)$ in place of $1/k^2$. The weight of a spinor's closed history with the link inside is the published one in a constant field, taken before the link's proper time is integrated [57]. With a body's mass m used only to cut the long histories, that closed history ends at $\ln(1/\lambda^2 m^2) - 2.019$ and the body's own line with the link on it at $\ln(1/\lambda^2 m^2) + \frac{1}{2}$. The two enter as twice the second less the first, $\ln(1/\lambda^2 m^2) + 3.019$, where minimal subtraction has $\ln(\mu^2/m^2) + \frac{15}{4}$; half the difference of the constants is the shift of the end, so the second-order logarithm of this part ends 0.365 e-folds below $\hbar c/\lambda$. This part is 107 per cent of light's second-order slope at the cell. It raises light's ask by 0.00035 and lowers $1/\alpha$ at rest by 0.0045. The histories in which the link that is read meets the exchanged link's own line are now worked out. Their weight has two parts. The first is the closed history with the link inside and the read link on both, at fixed proper times; summed the standard way it gives three tenths of the published coefficient. The second lies where the exchanged link's stretch is so long that the read field bends the link's own quanta: there the closed history is a point on the link's line and enters through its first-order value, and the link's line closes on itself. It gives the other seven tenths, and a weight written at fixed proper times cannot carry it. Carried to any number of dimensions, the two together are the published two-loop expression in every dimension [63]. Under the count the first part ends at $\ln(1/\lambda^2 m^2) - 1.495$ and the second adds no constant, because it lives at stretches far above one unit and its other factor is the closed history's first-order logarithm; minimal subtraction has $-\frac{8}{15}$ on the whole. So this part's logarithm ends 0.042 e-folds above $\hbar c/\lambda$ on spinor rings, and 0.91 e-folds above on the scalar's ring, which carries 1.6 per cent of it for light. It lowers light's ask by 0.000047 and colour's by 0.000049, and raises $1/\alpha$ at rest by 0.0006. The scalar's closed histories with another thing's link inside are worked out the same way: the published scalar weight in every dimension [57], the scalar's own line with the link on it at $\ln(1/\lambda^2 m^2) + \frac{3}{2}$ where minimal subtraction has $\frac{7}{3}$, and an end 0.408 e-folds below $\hbar c/\lambda$; light's ask rises by 0.000033 and $1/\alpha$ falls by 0.0004.

The top's closed history with the scalar inside is worked out in the same way, with the scalar in the exchanged link's place. The closed history is the quark's: one stretch as a member of the left-handed pair, the other as the right-handed top, joined at two places by the scalar's stretch. The scalar carries the difference of the two stretches' charges, so where the two differ

the link that is read meets the scalar's stretch as well. We derived the weight from the action, for stretches with charges q_1 and q_2 under the link that is read and a scalar with $q_1 - q_2$. It is a quadratic form in the two charges, $A\,(q_1^2 + q_2^2) + B\,q_1 q_2$, and at equal charges it is the weight of a neutral scalar. Three first-order logarithms stand beside it, each cut at a mass, and the second order moves each mass to the pole of its dressed line. The two stretches' own lines with the scalar on them take $\frac{4}{3}\,(q_1^2 + q_2^2)$ from the weight's logarithm, as a body's own line does when a link is exchanged. The scalar's own line with the quark's closed history on it adds $\frac{2}{3}\,(q_1 - q_2)^2$: the closed history adds to the scalar's kinetic term, so the pole of the scalar's line lies below its bare mass and the scalar's own first-order logarithm, cut there, is longer. The published two-loop running, $2\,(q_1^2 + q_2^2)$ in the weight's units with no product term [31], therefore asks $\frac{8}{3}\,(q_1^2 + q_2^2) + \frac{4}{3}\,q_1 q_2$ of the weight. Where a stretch of the quark and the scalar's stretch are both short the weight's integral converges only conditionally, and its value depends on the order. The count's order, the two meeting places anywhere along the closed history first and the scalar's stretch after, gives $\frac{8}{3}$ and $\frac{4}{3}$; the other order gives $-\frac{20}{3}$ in place of $\frac{4}{3}$. Carried to any number of dimensions, where a scalar without mass has no first-order logarithm, the weight has no product term in its pole, and with the stretches' own lines the pole is the published one. Summed over whole counts, with each stretch's own line at $\ln(1/\lambda^2 m^2) - \frac{1}{2}$ and the scalar's own line at $\ln(1/\lambda^2 m^2)$ with no constant, the part's logarithm ends at $\ln(1/\lambda^2 m^2) - 1.594 + 2.158\,\rho$, with $\rho = q_1 q_2/(q_1^2 + q_2^2)$, where minimal subtraction has $\ln(\mu^2/m^2) - \frac{5}{4} + 3\rho$. Half the difference of the constants is the shift of the end, so the part ends $0.172 + 0.421\,\rho$ e-folds below $\hbar c/\lambda$. Colour reads both stretches alike and the scalar not at all, $\rho = \frac{1}{2}$: 0.383 e-folds below, the value for a neutral scalar. The weak links read the left-handed stretch only, $\rho = 0$: 0.172 below. Hypercharge reads the stretches at $\frac{1}{6}$ and $\frac{2}{3}$ and the scalar at $\frac{1}{2}$, $\rho = \frac{4}{17}$: 0.271 below. Light's part is the weak links' and hypercharge's, weighed nine to seventeen, and ends 0.237 e-folds below. Said by electric charge, eight thirteenths of it is the top on both stretches with the neutral scalar and five thirteenths the bottom and the top with the charged scalar, which ends 0.004 below; the two weights are $q_1^2 + q_2^2$, $\frac{8}{9}$ and $\frac{5}{9}$. The part lowers light's ask by 0.000012 and colour's by 0.000009, and raises $1/\alpha$ at rest by 0.0002. A second derivation, made independently from the action, gives the same weight and the same sums over whole counts, and, with the same reading of the scalar's own line, the same constants.

One other count of this history is possible in form and is not taken: the quark's two stretches each counted as a line, from one, as the scalar's stretch is. It fails when computed. In the body's squared law the scalar closed at one place cancels the scalar between two places at no momentum only when the stretch between the two places is free; counted from one, it leaves the quark a squared mass of $(2\zeta(3) - \zeta(2))\,y^2/16\pi^2$ in the cell's unit, with y the top's coupling to the scalar, and the history then runs with $3.55\,(q_1^2 + q_2^2) - 4\,q_1 q_2$ in place of the published $2\,(q_1^2 + q_2^2)$. The count taken gives the quark no mass at any count and the published running for every pair of charges.

With every part computed, the second order's one line is for which of two equal stretches of the links' own quanta is the line: 0.00004 and 0.00007 in light's ask and 0.0006 and 0.0009 in $1/\alpha$, 0.0072 and 0.0073 in all with the rest of the error.

The count of the bodies' histories does not settle one kind of history, the one in which the links' own quanta meet one another. Three stretches of one field's quanta join at two places,

or two closed stretches touch at one, and none of them is another thing's. The record names none of them the line, but it fixes two things the count must reproduce. The first order has the links' own quanta run their logarithm the whole way to the cell, as the couplings confirm, so the count must give them no mass. And over long histories it must give the couplings' published two-loop running, whose coefficient from the gauge field's own quanta is $\frac{34}{3}C_A^2$ [59, 60]. A count that meets both takes the longest stretch as the line, summed plainly over its count from one; the others close the loop, at one over its count, with the places where the line leaves it anywhere along it. Where a stretch of the loop shrinks to nothing, so that two meeting places become one, the term this leaves is counted with the loop. Then the mass vanishes in any number of dimensions, not only in four, and the long histories give $\frac{34}{3}C_A^2$ exactly. The record does not state this count as a rule. It would if it said that, where a history of one field's quanta has two cycles, the loop counted at one over its count is the shortest cycle and the remaining stretch is the line. We do not assume that; the count rests on the two facts. Counting such a two-cycle history once, without separating the longest stretch as the line, fails under either reading of its gauge weight: under the textbook gauge weight it leaves the links a mass proportional to $-8\,I_1 I_2$, where $I_2 = -\frac{5}{3}$ in every gauge and I_1 vanishes only at the unlawful gauge parameter $\xi = -\frac{1}{3}$; taking the meeting rule whole, with the longest stretch serving as the meeting link, removes the mass exactly, but the one-count sum then runs at $-\frac{76}{3} = \frac{19}{17}(-\frac{68}{3})$ (converged to 2×10^{-10}, where $-\frac{68}{3} = -\frac{1}{2} \cdot \frac{34}{3}C_A^2$ at $C_A = 2$ is the weight's unit for the two-loop running), nineteen seventeenths of the published coefficient. The longest-stretch split is therefore required for the two-loop running as well as for the absence of a mass, and the two-statement placement below sits downstream of that split.

The weight is the published one for the gauge field's own two-loop history in a constant field, with its ghosts, expanded to second order in the field [61, 62]; we derived its graphs again from the action in four to eight dimensions, and they agree term by term. Recomputing one pole term of that weight, we find $+1/(12\epsilon)$, from an integral of exactly $1/6$, where [61] prints $-1/(12\epsilon)$; the paper's own total needs the + sign, and [62] prints it. With the loop's total proper time cut at $1/\mu_0^2$ on both sides, minimal subtraction has $\ln(\mu^2/\mu_0^2) - 0.095$, and the count's logarithm is $\ln(1/\lambda^2\mu_0^2) - 0.230$ or $\ln(1/\lambda^2\mu_0^2) + 0.661$, as the two statements below have it. Half the difference of the constants is the shift of the end, so this part's second-order logarithm ends 0.07 e-folds below $\hbar c/\lambda$ or 0.38 above. It moves light's ask by -0.000063 or $+0.000353$ and colour's by -0.000133 or $+0.000741$, the first toward light's value at the cell and the second away from it.

Here the count leaves one thing open. Moving a stretch's own term within the loop leaves a term at each end of the stretch's range: where the stretch shrinks to nothing, so that two meeting places become one, and where it grows as long as the line, beyond which it is itself the line. The record has one statement on a move between counts: marking one recognition of a closed history opens it, and the change lands on the history with its ends joined, the step itself counted in full. That fixes the first end. It says nothing of the second, which exists only because the line is the longest stretch. Two statements would settle it. The first: "A stretch's own term, moved by parts in that stretch's proper time with the other loop stretch and the line held fixed, moves only while the stretch is shorter than the line. Where it reaches the line's length the move ends, and its end is counted in full, once, as a term of the history there." The second: "A stretch's own term, moved by parts in that stretch's proper time with the other two held fixed, moves over the whole of that stretch's proper time. Where the stretch is

the longest of the three, and so is the line, the moved term is summed over the line's whole counts." The record makes neither. Both give no mass and the published running exactly, and they give this part's constant −0.068 and +0.378 e-folds. Within each, which of two equal stretches is the line, where a stretch of the loop is as long as the line, is open as well; it moves the constant by ±0.048 and ±0.073, carried in the error. We therefore give the result for each statement.

Structural anatomy and obstruction on the two-cycle placement. Three exact structural results (code archive, `open_premises_analysis/`) sharpen the gap between the two statements. Write δ_5 for this part's e-fold shift ($\delta_5 = -0.0678$ under the first statement and +0.37779 under the second), T_X and T_Z for the moved loop stretch and longest line stretch, $b = T_{\text{line}}/T_{\text{loop}}$ for the line-to-loop proper-time ratio, and $\mathscr{L}[j] = \sum_{N\geq 1} \frac{1}{N}\big[\frac{1}{N}\sum_{n\geq 1} j(n/N) - \int_0^\infty j\,db\big]$ for the whole-count ring offset defined in the appendix. First, the two statements are endpoints of a one-parameter continuum-valid family $\gamma \in [1,\infty]$ obtained by stopping the stretch move at $T_X \leq \gamma T_Z$: every $\gamma \in [1,\infty]$ preserves zero link mass in all dimensions, finite integral $J_\gamma = -\frac{32}{3}$ and long-line pole −12 (for every finite γ, shifting to −14 and $-\frac{26}{3}$ at $\gamma = \infty$), and invariant two-loop coefficient $-\frac{68}{3}$, with $\lim_{\gamma\to\infty}\delta_5(\gamma) = +0.37779$ (the second statement). Every finite $\gamma \geq 1$ creates two strictly positive artificial boundary jumps in the moved density,

$$j_{\text{bnd},\gamma}(\gamma/2^+) = \frac{16(10\gamma^3 + 15\gamma^2 + 10\gamma + 4)}{3\gamma(2\gamma+1)^3} > 0, \qquad j_{\text{bnd},\gamma}(\gamma^-) = \frac{4(5\gamma^2 + 6\gamma + 5)}{3\gamma^3} > 0$$

($\frac{208}{27}$ and $\frac{64}{3}$ at $\gamma = 1$); *only the second statement ($\gamma = \infty$) eliminates both artificial boundary discontinuities.* By Euler–Maclaurin decomposition, 86.9 per cent (+0.38704 of +0.44556 e-folds, $\Delta\mathscr{L} = -\frac{\zeta(2)}{2}\cdot\frac{64}{3} = -17.546$) of the difference between the two statements comes from the first statement placing full weight $+\frac{64}{3}$ on the artificial far-end tie at $b = 1$ (sitting at the peak of a narrow $\Delta\gamma \sim 0.1$ cusp, with $\lim_{\gamma\to 1^+}\delta_5(\gamma) = -0.0328$ as $b = 1$ moves strictly inside the boundary support, versus +0.3193 for the $\gamma = 1$ midpoint), and 13.1 per cent (+0.05851 e-folds unrounded) is the bulk slope defect. Second, continuum proper-time integrands under whole-count loop-plus-line foliation cannot fix δ_5 uniquely: on an elementary compactly supported degree-−2 quadratic tent with identical continuum integral $-\frac{7}{24}$, moving a total derivative across the longest-stretch seam $T_X = T_Z$ shifts $\mathscr{L}[\cdot]$ strictly positively on every ring $N \geq 1$, summing to the exact closed form $+\frac{15}{128}\zeta(3) > 0$ under midpoint jumps and $+\frac{15}{128}\zeta(3) + \frac{1}{2}\zeta(2) > 0$ under the first statement's full far-end weight. Third, direct discrete alternatives that avoid choosing γ either break zero link mass or cluster around the second statement: in the shared-link kernel below, the gluon kernel agrees term-for-term with the un-pinched two-loop kernel in all D and the full gluon-plus-ghost kernel matches identically after S_3 stretch symmetrization, scaling as $\kappa^{4-D} \to \kappa^0$ at $D = 4$ (carrying no scale information for δ_5, while direct continuum-loop discretization without the stretch move leaves mass coefficient $-\frac{20}{3}$); independent three-stretch discrete summation violates one-loop Ward closure by $-2\zeta(3) \neq 0$ and leaves exact two-loop link mass $-\frac{20}{3}\zeta(2) + \frac{80}{3}\zeta(3) \approx +21.089$ (un-pinched) and $+\frac{40}{3}\zeta(3) \approx +16.027$ (pinched with discrete contacts); and discrete telescoping along the line hits an exact non-integrable $-\frac{20}{3\nu^2}$ contact pole at $N = 1$ (where ν is the short-stretch split inside the loop) for backward and symmetric differences, while the unique all-N finite forward telescope gives $\delta_5 = +0.33815$ ($1/\alpha = 137.0198$), with clamped symmetric (+0.38559), hybrid

(+0.42074) and clamped backward (+0.43303) all clustering around the second statement (+0.37779) and none near the first (−0.0678). Because continuum foliation cannot settle δ_5 unconditionally without a native discrete closure-preserving two-cycle kernel on the 24-bond lattice, this paper continues to report both headline values side by side, noting plainly that every discontinuity-free continuum and finite discrete-telescoping criterion selects the second statement (137.0193 ± 0.0073).

Native shared-link derivation of the pure-link two-loop coefficient $34/3$**.** The published pure-gauge two-loop coefficient $A = \frac{34}{3}$ cited above [59, 60] can now be computed directly from the shared-link three-stretch kernel rather than borrowed through a supersymmetric identity. Three stretches of heat lengths ℓ_1, ℓ_2, ℓ_3 join two events in a constant background turn in $D = 2 + n$ directions, with the link's own-turn factor at rate $r_t = 2$ and the relabelling (Faddeev–Popov ghost) pair alongside the three-link cubic rule. Cutting all three stretches symmetrically with a uniform cut on the short pair's total length $\rho = \ell_j + \ell_k > \delta$ on each face where stretch i is longest, adding the dimension-continued corner term $(-2\,\partial_D a_2)|_{D=4}$ (with a_2 the $\rho^{-D/2}$ corner coefficient) and one single-stretch carry mechanism (checked on the hopping pair, i.e., a complex scalar, to give $C_s = 4$ and $B_s = \frac{2}{3}$, alongside the matter-hand Weyl-fermion constants $B = \frac{10}{3}$ and $C_b = 2$), yields the exact breakdown:

class	finite part	corner term	link carry	relabelling carry	sum
three link stretches	21/4	−25/4	21/2	—	19/2
relabelling pair + link	3/4	−5/12	7/6	1/3	11/6
total	6	−20/3	35/3	1/3	34/3

Every $\ln 3$ cancels between faces. Grouped by mechanism, $A = 6 + 6 - \frac{2}{3}$: finite parts 6, carries less their dimension halves $6 = \frac{5}{3} \cdot \frac{7}{2} + \frac{1}{2} \cdot \frac{1}{3}$ (the link's first-step coefficient $\frac{5}{3}$ times half its carry response 7, giving $\frac{35}{6}$ against the table's full carry $\frac{35}{3}$, plus the relabelling stretch's $\frac{1}{2}$ times half its carry response $\frac{2}{3}$, giving $\frac{1}{6}$ against $\frac{1}{3}$), and the own-turn dimension term $-\frac{2}{3}$. Leaving the link's own-turn rate r_t free gives the quadratic polynomial $A(r_t) = \frac{49}{12} r_t^2 - \frac{1}{2} r_t - 4$ ($\frac{49}{3} - 1 - 4 = \frac{34}{3}$ at $r_t = 2$). Including the four-link meeting term at one event adds +24 to the second-order corner logarithm, cancelling the three-stretch −24 exactly; with r_t free the total corner logarithm is proportional to $r_t(r_t - 2)$, vanishing uniquely at the physical link rate $r_t = 2$ among nonzero rates. Together with $B = \frac{10}{3}$ and $C_b = 2$, this closes the two-loop slope ratios $\beta_1/\beta_0^2 = \frac{102}{121}$ (pure non-Abelian gauge, any simple non-Abelian gauge group) and $\frac{26}{49}$ (six-flavour QCD) natively, making the relation $A = 6 + B + C_b$ a consequence rather than a borrowed input.

Three analytical upgrades narrow the scope limits of this derivation (code archive, `pure_link_two_loop_A/`). First, all seven distinct face classes across the thirty-six evaluated faces are now established analytically by exact ray integration from the corner ($y = tx$ and $x = ty$, $t \in [0, 1]$) into elementary closed-form primitives in $(1 + t)^{-1}$, $(1 + 2t)^{-1}$, $\ln(1 + t)$ and $\ln(1 + 2t)$, leaving no unevaluated transcendental integral (only $\ln 3$ appears in individual faces and cancels identically in $A = \frac{34}{3}$), while the second-order numerators agree

identically across two independently written engines in every dimension D and match direct numerical quadrature to 1.12×10^{-10}. Second, evaluating the two-face four-link meeting term $(c_{12}/(\ell\ell') + c_1/\ell'^2 + c_2/\ell^2$ on $0 < \ell' < \ell = 1$, with $(c_{12}, c_1, c_2) = (12, 10, 10))$ across three short-stretch regulators proves that its finite part is identically 0 under both a sharp cut $\ell' > \varepsilon_{\text{cut}}$ and a power-law regulator $(\ell')^\epsilon$ (with log-log count 12 in every dimension, hence zero direction-count term), whereas under a smooth profile $u_{\text{sm}}^2/(1+u_{\text{sm}})^2$ $(u_{\text{sm}} = \ell'/\varepsilon_{\text{cut}})$ its finite part is -24 (not zero) and is offset by the opposite shift of the charged faces; under cut rescaling $\varepsilon_{\text{cut}} \mapsto s_{\text{cut}}\varepsilon_{\text{cut}}$, the three-stretch charged faces and the two-face meeting term shift by $+24 \ln s_{\text{cut}}$ and $-24 \ln s_{\text{cut}}$, respectively, leaving $A = \frac{34}{3}$ strictly cut-scale invariant for every $s_{\text{cut}} > 0$. Third, running the exact same unchanged uniform-cut reader on the Weyl-fermion matter hand with D free and stretch-charge pair (q_i, q_j) gives, on the first run, $S_{\text{hand}}(1,1) = -\frac{2}{3} + 0 + \frac{8}{3} = 2 = C_b$ and $S_{\text{hand}}(1,0) = \frac{8}{3} - \frac{13}{3} + 6 = \frac{13}{3}$, hence $B = S_{\text{hand}}(1,0) - \frac{1}{2}S_{\text{hand}}(1,1) = \frac{10}{3}$ (for $(1,0)$, with corner logs $16 + 0 + 16 = 32$ cancelling twice the hand's meeting coefficient 16, and every $\ln 3$ cancelling identically). One unchanged reader therefore yields all five two-loop constants $(A = \frac{34}{3}, C_s = 4, B_s = \frac{2}{3}, C_b = 2, B = \frac{10}{3})$ and closes $\frac{102}{121}$ and $\frac{26}{49}$ natively. Four limits remain plainly in scope: (i) the second-order numerators agree across two independent engines in all D but come from inherited long-wave continuum kernel rules rather than being derived from the 24-bond cell; (ii) for the smooth profile $u_{\text{sm}}^2/(1+u_{\text{sm}})^2$, cancellation against individual charged faces is checked numerically (exact analytically for the rescaled sharp-cut family $s_{\text{cut}}\varepsilon_{\text{cut}}$); (iii) for the matter hand, the meeting-term coefficient (16) and fixed four-component trace are inherited inputs; and (iv) as proved above, this fixes the universal two-loop running coefficient $\frac{34}{3}$, not the finite second-order matching constant separating 137.0246 from 137.0193.

The second order's constants in one place. Each part's second-order logarithm ends δ e-folds above $\hbar c/\lambda$, or below it where δ is negative, and δ is half the difference of the part's two constants, the count's less minimal subtraction's. An end that lies δ e-folds above moves light's ask by the part's slope times δ. Light's ask is its first-order value plus its six shifts: under the first statement $8.32381334 + 0.00026390 = 8.32407724$, printed as 8.324077, and under the second $8.32381334 + 0.00067978 = 8.32449312$, printed as 8.324493. The value of $1/\alpha$ at rest is the measured value less light's distance from $8K$ over the slope of section 3: $137.035999 - (8.324077 - 8.323168)/0.0794553 = 137.0246$ under the first statement and $137.035999 - (8.324493 - 8.323168)/0.0794553 = 137.0193$ under the second. The appendix rebuilds both values part by part from numbers printed by the code archive. The second and third columns are the constants under the count and under minimal subtraction. Slopes are light's, for each e-fold of the end, and shifts are of light's ask; both are in units of 10^{-6}; the first slope is taken at four figures. The count's constants are built as follows. Spinor histories with another thing's link inside: twice the body's own line, $\frac{1}{2}$, less the closed history, -2.01927. The read link on the exchanged link's own line: three tenths of -1.49476 on spinor rings; on the scalar's ring the part at fixed proper times is $-\frac{1}{4}$ of a whole of $\frac{1}{3}$ and ends at $+0.24256$, so the constant is three times that. Scalar histories with another thing's link inside: the closed history gives $\frac{3}{2}\ln(1/\lambda^2 m^2) - 1.88157$ and the scalar's own line with the link on it $\frac{1}{2}[\ln(1/\lambda^2 m^2) + \frac{3}{2}]$, together $2[\ln(1/\lambda^2 m^2) - 0.56579]$, where minimal subtraction has $2[\ln(\mu^2/m^2) + \frac{1}{4}]$, from $-\frac{2}{3}$ and $\frac{1}{2} \cdot \frac{7}{3}$. The top's closed history with the scalar inside: for stretches with charges q_1 and q_2

part	count	subtraction	δ	slope	shift
spinor histories, another thing's link inside	+3.019	$\frac{15}{4}$	−0.36536	−969.0	+354.03
the read link on the exchanged link's own line, spinor rings	−0.448	$-\frac{8}{15}$	+0.04245	−823.52	−34.96
the same on the scalar's ring	+0.728	$-\frac{13}{12}$	+0.90551	−13.73	−12.43
scalar histories, another thing's link inside	−0.566	$\frac{1}{4}$	−0.40789	−79.86	+32.57
the top's closed history with the scalar inside	−1.262	−0.788	−0.23683	+50.81	−12.03
the links' own quanta, first statement	−0.230	−0.095	−0.0678	+933.33	−63.28
the links' own quanta, second statement	+0.661	−0.095	+0.37779	+933.33	+352.60
all six, first statement					+263.90
all six, second statement					+679.78

under the link that is read and $\rho = q_1 q_2/(q_1^2 + q_2^2)$, the weight's two sums over whole counts are −3.85495 and +4.31619; with each stretch's own line at $\ln(1/\lambda^2 m^2) - \frac{1}{2}$ and the scalar's own line with no constant they give $2\,(q_1^2 + q_2^2)\,[\ln(1/\lambda^2 m^2) - 1.59414 + 2.15810\,\rho]$, where minimal subtraction has $2\,(q_1^2 + q_2^2)\,[\ln(\mu^2/m^2) - \frac{5}{4} + 3\rho]$. Light's row weighs the weak links ($\rho = 0$) and hypercharge ($\rho = \frac{4}{17}$) nine to seventeen. The links' own quanta: under the first statement $\frac{8}{17}$ of −0.48959, and under the second $\frac{21}{34}$ of 1.06973.

One unit is one recognition: each of the eight columns, four accounts with a debit and a credit, posted once. The four debits are an orthonormal frame of bonds, and so are the four credits, so one recognition's second moment is exactly twice the round one and its proper time is exactly λ^2: the logarithms end at $\hbar c/\lambda$. Each posting weighs a displacement of x entries by $e^{-x^2/2}$ on each account. That number has been reached in three ways, which are one number and not three supports. It is the least state of an action that weighs a count and its turn alike, the form the ledger's posting law has; the function is its own transform, with the turn in radians. It is the cost's own curvature at balance. And it is the pair cost of one golden entry: read on its pair a comparison z costs $|z - 1/z|^2/2$, half the squared distance between its two sides, and the golden entry's sides differ by exactly one. The principle is older than the comparison at the cell, and so is the number. The record prices a delta of Δ postings on one side of one account at $\cosh(q\Delta) - 1$, with q the strain one posting presents, and had left q open: it names values for it, each fixed by one definition, and derives none. Among them is the half adopted here: where a debit posting with a credit posting costs one, at $q = 2\ln\phi$ with $\phi = (1 + \sqrt{5})/2$, one posting presents the ratio ϕ^2 and costs $J(\phi^2) = \frac{1}{2}$. What was chosen with the end in view is that a displacement of one entry in a history that leaves no record is costed as that posting. The couplings then decide what the record had left open. Read back from the three couplings, the strain one posting presents is 0.9627 ± 0.0068 and 0.9607 ± 0.0068, against $2\ln\phi = 0.9624$.

The record's own convention, $q = 1$, at which one post costs $\cosh 1 - 1 = 0.543$, would end the logarithms at $1.042\,\hbar c/\lambda$, 2.5 and 1.9 errors from the end light and colour ask and 5.4 and 5.7 from the end the three couplings ask. The values it orders beside these two, one golden rung, one bit and one coin a posting, miss by 45 errors or more. A debit post with a credit post at one golden rung each, $2J(\phi) = 0.236$, taken for one entry's cost, would end them at $0.69\,\hbar c/\lambda$: 49 errors off, and more than eight in the combination of light and colour from which the dressing and the count's four both drop out. With the turn counted in whole cycles the same principle would end the logarithms at $2.51\,\hbar c/\lambda$, and with the square of the least state at $1.41\,\hbar c/\lambda$; the measured end excludes both, by more than forty errors. The whole-cycle form is the posting law's own: the law states no kernel, and its least state at fixed norm, worked out here, is $e^{-\pi x^2}$ on each coordinate, repeated around the law's three phases, with x in the law's own extent; the kernel adopted here is the same function with the bond at $1/\sqrt{2\pi}$ of that extent. The radian, the bond as the posting's extent and the least state itself were chosen with the end in view. Read back from the measured couplings, one entry costs 0.515 ± 0.011 and 0.522 ± 0.011; the entry read on one side only (0.118) is 37 errors off, and four times that cost at small size (0.472) 4.0 and 4.6.

The measured couplings put the end at $(0.99 \pm 0.04)\,\hbar c/\lambda$ and $(0.98 \pm 0.04)\,\hbar c/\lambda$ from light less twice colour, which contains no dressing, and at $(1.015 \pm 0.011)\,\hbar c/\lambda$ and $(1.022 \pm 0.011)\,\hbar c/\lambda$ from light and colour with the dressing: 7.76 ± 0.16 and 7.66 ± 0.16 postings of bond size to a unit, where the ledger has eight, and a unit proper time of 0.970 ± 0.020 and 0.957 ± 0.020 in units of λ^2. Seven is 5.0 and 4.3 errors away and nine 7.1 and 7.7, counted on the logarithm of the end, where the error is the same on both sides; the ± 0.16 is that error carried to the postings at their centre. The weak share hardly involves the dressing, and given the form of the stay (section 5) it reads the end by itself: $(0.999 \pm 0.008)\,\hbar c/\lambda$ and $(0.995 \pm 0.008)\,\hbar c/\lambda$. The three couplings together put the end at $(1.000 \pm 0.008)\,\hbar c/\lambda$ and $(0.998 \pm 0.008)\,\hbar c/\lambda$: a unit of 0.999 ± 0.015 and 1.004 ± 0.015 in units of λ^2, 8.00 ± 0.12 and 8.03 ± 0.12 postings, with seven and nine more than seven errors away, and one entry's cost at 0.500 ± 0.008 and 0.498 ± 0.008. Left free, the dressing is fixed by the same three couplings at 1.040513 ± 0.000054 and 1.040586 ± 0.000055; the computed 1.040396 lies 2.2 and 3.4 of that error below. The coefficient of section 2.4 states the same shortfall: with the end held at the cell, the three couplings locate it at 0.9971 ± 0.0013 and 0.9954 ± 0.0013, 2.1 and 3.4 errors below one. With no determination of the hadronic input used at all, the couplings still put the unit at 1.035 ± 0.023 and 1.063 ± 0.024 and ask the input 0.02798 ± 0.00005 and 0.02807 ± 0.00005. These figures are the three landings read as an end, not a further test of the closure, and the share's part of them stands or falls with the form of the stay. One neighbour that is not a whole number of postings, the exact cost read as an amplitude (an end at $1.0305\,\hbar c/\lambda$), lands on light and colour, so those two fix the unit to two per cent, not to the integer by themselves; with the weak share in, it does not land ($\chi^2 = 20.1$ and 29.6 for three; every chi-square in this paper counts the couplings' correlations). That neighbour, and the one that weighs a posting by the exact cost of its length as a weight (an end at $1.290\,\hbar c/\lambda$), are excluded without a measurement: a step weighed by the exact cost of its length obeys no law of comparison in a plane, whatever product term it carries, so only the plain law weighs where a posting goes. The first also puts the cost in the amplitude, which the carrier excludes (section 2.4). Minimal subtraction at the cell misses light by more than eight errors under the first statement and nine under the second, and reading the coupling at the zone's edge π/λ by more than ninety. A cut on each

line of a loop, which is what a regulator of fields does, would give scalars, spinors and vectors different ends; the measured couplings ask for one end for every spin: the one combination of the three couplings that does not involve the end stands 2.2 and 3.4 errors from zero, and the cut lies 40 and 38 errors away. That 2.2 and 3.4 measures the same shortfall as the 2.2 and 3.4 of the dressing and the 2.1 and 3.4 of the coefficient above. The inside of a history that leaves no record is read as not on the record's places: lattice places inside such a history would be refused by gravity's two bounds, the speed of its waves and the absence of a mass for the graviton, and no coupling enters that.

Gravity at the cell belongs to this matching. In units of the cell's area Newton's constant is $1/(4\ln 2) = 0.36$, which is not small. If the strain's own histories were summed like a field's, light would move by tens of per cent (by a factor 1.41 in a cutoff scheme [23]). Within two errors the closure leaves room for a relative addition between -0.00002 and $+0.00024$ under the first statement; under the second, light's shortfall is more than two errors, and an addition between $+0.00003$ and $+0.00029$ would bring it within two. The result therefore rests on the fourth of the statements listed below: the strain is the record's own geometry, sourced and never summed.

Four statements carry the matching at first order and are not yet derived from the ledger's foundation: that the count of an unrecorded history is summed, that a posting which leaves no record is in the least state of the equal-weight action (which sets the unit), that the links' own turns are not summed as weights at the cell (section 2.4), and that gravity's strain is never summed. With them the first-order matching has no free number. The table of section 8 marks four more as readings: that the inside of a history that leaves no record is not on the record's places, on which the first order rests too; and, at second order, that an exchanged link meets a history as the link that is read does, the count where the links' own quanta meet one another, and where a moved term is counted, which the record leaves open and which gives the two values.

3 The carry from rest to the cell

What the measured couplings are at the cell is a Standard Model calculation, and we use it as it stands. The state of the Standard Model at 200 GeV is taken from the interpolation formulas of Alam and Martin [28], which match the on-shell measured inputs (the fine-structure constant, the Fermi constant, the masses of the Z, the top quark and the Higgs boson, the strong coupling, the quark and lepton masses, and $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$) to the Lagrangian couplings at three loops and beyond. From there the three-loop renormalization-group equations [31, 32, 33] run every coupling to the cell. Three effects beyond that running are known and are applied to light's ask: the complete three-loop running of the Yukawa couplings and the Higgs self-coupling, as the reference code's own published running shows from a shared start ($+0.000005$; [29]); the four-loop gauge terms ($+0.00005$; [34]); and the top quark's decoupling from light at order $\alpha\alpha_s^2$, which the two-loop matching at the Z [30] leaves out (-0.00006; [36]). Converted to pure logarithms, with the 2026 electroweak inputs, $\alpha_s(M_Z) = 0.1180(9)$ and the 2026 lattice value of the hadronic input, the measured couplings ask at the cell:

	light, $1/e^2$	colour, $1/g_3^2$	weak share, g^{-2}/e^{-2}	
Inputs: $1/\alpha(0) = 137.035999$, $G_F = 1.1663787 \times 10^{-5}\,\mathrm{GeV}^{-2}$, $M_Z = 91.1876\,\mathrm{GeV}$ and $\alpha_s(M_Z) = 0.1180$; from the Particle Data Group's 2026 Listings (weighted averages) $M_t = 172.60$ and $M_h = 125.13\,\mathrm{GeV}$ and the quark masses $m_b(m_b) = 4.1859$ and $m_c(m_c) = 1.2729\,\mathrm{GeV}$ and, at 2 GeV, $m_s = 92.9$, $m_u = 2.16$ and $m_d = 4.70\,\mathrm{MeV}$; the lepton masses, which enter the state at 200 GeV only through their Yukawa couplings; and $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027821$, the 2026 lattice determination. M_W does not enter: Alam and Martin's state takes G_F and M_Z instead.				
At 200 GeV (Alam and Martin), minimal subtraction	10.1538782	0.7522851	0.2354026	
run to the cell, three loops	−1.8258387	+3.4176250	+0.2323621	
At the cell, minimal subtraction	8.3280396	4.1699101	0.4677647	
pure logarithms: less $C_A/(48\pi^2)$	−0.0042217	−0.0063326	−0.0002699	
the complete three-loop running	+0.0000052	+0.0000080	+0.0000000	
the four-loop gauge terms	+0.0000500	−0.0000010	−0.0000006	
the top quark's decoupling at order $\alpha\alpha_s^2$	−0.0000597	+0.0000000	+0.0000041	
First order	8.3238133	4.1635846	0.4674982	
second order: the bodies' closed histories with a link inside	+0.0003540	+0.0001584	−0.0000008	
second order: the read link meets the exchanged link's own line, spinor rings (+0.042)	−0.0000350	−0.0000490	−0.0000022	
second order: the same on the scalar ring (+0.906)	−0.0000124	+0.0000000	−0.0000008	
second order: scalar rings with another thing's link inside (−0.408)	+0.0000326	+0.0000000	+0.0000001	
second order: the top with the scalar (−0.237)	−0.0000120	−0.0000090	+0.0000003	
second order: the links' own quanta, kept in the loop (−0.068)	−0.0000633	−0.0001330	−0.0000040	across
Asked at the cell, kept in the loop	8.3240773	4.1635520	0.4674908	
second order: the links' own quanta, carried into the line (+0.378)	+0.0003526	+0.0007412	+0.0000226	across
Asked at the cell, carried into the line	8.3244932	4.1644262	0.4675174	
Errors				
the hadronic input (±0.000049)	0.0005335	0.0000000	0.0000364	within
the strong coupling (±0.0009)	0.0001953	0.0049751	0.0000050	within
the top mass (±0.27 GeV)	0.0000580	0.0000263	0.0000112	within
the Higgs mass (±0.11 GeV)	0.0000000	0.0000001	0.0000005	within
the Z mass (±0.0021 GeV)	0.0000036	0.0000002	0.0000186	within
the reference state's last revision	0.0000150	0.0000000	0.0000397	within
the matching at the Z after its three-loop top term	0.0000300	0.0000000	0.0000020	within
the reference state's interpolation formulas	0.0000170	0.0000000	0.0000010	within
the running beyond four loops	0.0000100	0.0000100	0.0000010	within
the dressing (±0.000002 in K)	0.0000160	0.0000080	0.0000000	within
left open by the count: the links' own quanta, which equal stretch is the line (0.048), kept in the loop	0.0000448	0.0000942	0.0000029	across
All lines, kept in the loop	0.0005744	0.0049761	0.0000584	
left open by the count: the links' own quanta, which equal stretch is the line (0.073), carried into the line	0.0000677	0.0001424	0.0000043	across
All lines, carried into the line	0.0005766	0.0049772	0.0000585	

Table 1. The carry from rest to the cell for light, colour and the weak share. Under the first statement of section 2.5 the links' own quanta are kept in the loop; under the second they are carried into the line; only that row and its line in the error differ. Each total error is the quadrature sum of the lines above it. The last column marks each line of the error, and each part the two statements set differently, as within one reading set (the statement and which equal stretch is the line both fixed) or across readings.

	first statement asked at the cell	error	second statement asked at the cell	error
light, $1/e^2$	8.324077	0.000574	8.324493	0.000577
colour, $1/g_3^2$	4.163552	0.004976	4.164426	0.004977
weak share, g^{-2}/e^{-2}	0.4674908	0.0000584	0.4675174	0.0000585

These are the asks with the matching's second order, every part of it computed. At first order they are 8.323813 ± 0.000573, 4.163585 ± 0.004975 and 0.4674982 ± 0.0000584; the second order moves them by $+0.000264$, -0.000033 and -0.0000075 under the first statement and by $+0.000680$, $+0.000842$ and $+0.0000191$ under the second, and the line for what the count leaves open, which of two equal stretches of the links' own quanta is the line, adds 0.000045, 0.000094 and 0.0000029 to the errors in quadrature under the first statement, and 0.000068, 0.000142 and 0.0000043 under the second (section 2.5). Those lines move the three together: with them the correlations are $+0.34$ between light and colour, -0.61 and -0.60 between light and the share and -0.09 and -0.08 between colour and the share. With these correlations the three couplings' chi-square is 4.70 under the first statement and 11.94 under the second.

Table 1 gives the carry step by step: the measured inputs, the couplings at 200 GeV, the run to the cell and each correction, the first-order asks, each part of the second order, and the asks at the cell under each of the two statements of section 2.5, with every line of their errors. From its rows a reader rebuilds light's ask. Within one reading set (the statement and which equal stretch is the line both fixed), light's error is that of the carry's lines alone, its first-order error 0.000573, under either statement (0.0072 in α^{-1}). Across readings, the line for what the count leaves open adds 0.000045 or 0.000068 to it in quadrature, giving 0.000574 and 0.000577; the two statements, whose difference neither statement's error contains, set light's ask 0.000416 apart, 0.72 of the first statement's error.

Light's error has these parts: the hadronic input (0.000534), the strong coupling (0.000195), the top mass (0.000058), and, together 0.00004, the matching of the reference state (its last revision moved light by 0.000015, its interpolation formulas by at most 0.000017), what remains of the matching at the Z after its three-loop top term (0.00003), and the running beyond four loops (0.00001). Kept as version 1.0 kept it, with the revision of the reference state added in quadrature over the two couplings, a line of 0.000573 that equals light's first-order error to the digits printed, light's carry error would be 0.000810 and, with the matching's line for what its count leaves open (section 2.5), light would sit at -1.12 and -1.63 instead of -1.58 and -2.30, with $\alpha^{-1} = 137.0246 \pm 0.0102$ and 137.0193 ± 0.0102 and the input asked 0.027905 ± 0.000075 and 0.027943 ± 0.000075; the value does not move and the conclusion does not depend on the choice. We use the signed step because the revision changed the electroweak scale, which moves g and g' in opposite directions (by -7 and $+2$ parts in 10^5) and leaves light, which is fixed by α and the hadronic input, almost unchanged ($+0.000015$), as a change of the Z mass does (0.000004); the matching of light itself, which both versions share, is carried separately (0.00003). The weak share's error keeps that revision whole, because a change of the electroweak scale moves the split even when it leaves the sum alone. Colour's error is almost entirely the strong coupling's. The series is short: from one loop to two the running moves light by -0.0531, from two to three by -0.0015, and the four-loop terms by $+0.00005$, opposite in sign and a thirtieth of the three-loop term [35]. A series whose next term is computed, and is a thirtieth of the last, does not have the last term as its error.

Four-loop three-coupling check and low-end electroweak reconciliation. To verify that neither four-loop running nor the low-end electroweak matching scheme hides a drift in the three-coupling closure, an independent check in the code archive (`four_loop_three_coupling/`) runs all three couplings at first-order cell matching (pure logarithms, $-C_A/(48\pi^2)$) on the 2024 Review of Particle Physics inputs [15] ($\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02783(6)$, $M_t = 172.57(29)$ GeV, $M_h = 125.20(11)$ GeV, $\alpha_s(M_Z) = 0.1180(9)$, $K = 1.040399$, distinct from Table 1's second-order carry on the 2026 Mainz lattice input; all pulls below follow the paper's uniform sign convention, ledger value less asked value divided by error). Three published two-loop electroweak determinations give $\sin^2\hat\theta_W(M_Z) = 0.231500$ (Martin and Robertson, pure $\overline{\rm MS}$ [29]), 0.231533 (Martin, adding three-loop QCD terms in M_W and M_Z [65]) and 0.231587 (Degrassi, Gambino and Giardino, hybrid [64], which puts the weak row at -2.22 errors at three loops). Fitting the pure $\overline{\rm MS}$ program `SMDR` v1.3, which adds leading three-loop corrections to the Fermi constant [66], directly on the 2024 inputs gives $\sin^2\hat\theta_W(M_Z) = 0.2315237$ and $M_W = 80.35022$ GeV (within 1.0 MeV of the on-shell prediction 80.35122 GeV [67], and stable to 2×10^{-6} across renormalization scales from 91 to 250 GeV). Swapping *only* the Fermi-constant relation $1+\Delta\hat r_W$ in that pure fit for the hybrid expression moves the share to 0.2315848, reproducing 97 per cent of the gap to 0.2315868: the W–Z mass splitting agrees to 7 parts in 10^6, tadpoles drop out identically, and the Fermi remainder differs by 1.84×10^{-4}. That isolates the cause of the low-end split without refuting the hybrid formula, since no complete three-loop electroweak Fermi relation exists.

Adding the complete four-loop Standard Model gauge β functions [34] (all 51 lower-order monomials checked against the three-loop polynomials; four-loop single-coupling g_3^9, g_2^9, g_1^9 Standard Model coefficients -2472.28, -627.3 and -120.7; Runge–Kutta step-halving error below 10^{-13}) shifts the three asked cell stiffnesses by -0.0000005, $+0.0000162$ and $+0.0000282$, that is 0.00, +0.036 and +0.043 (at most 0.044) of the registered three-loop row errors (0.005012, 0.000443 and 0.000649, which include the review's ±0.00004 theory error on the share; their sum +0.0000443 for light differs from Table 1's +0.0000500 by 0.0000056, 0.01 errors or +0.00007 in α^{-1}, changing no printed digit). The resulting four-loop pulls (ledger less asked, in row errors) are:

low-end determination (2024 inputs)	$\sin^2\hat\theta_W(M_Z)$	colour	weak	hypercharge
Martin–Robertson 2019 (strict two-loop pure $\overline{\rm MS}$)	0.231500	−0.42	−0.26	−0.70
Martin 2022 (+ three-loop QCD in M_W, M_Z)	0.231533	−0.42	−1.02	−0.20
2025 `SMDR` v1.3 fit (+ three-loop Fermi terms)	0.231524	−0.40	−0.81	−0.35
Degrassi–Gambino–Giardino 2015 (hybrid)	0.231587	−0.42	−2.26	+0.64

Over matching scales from 91 to 250 GeV the 2025 pure fit stays inside one error on every row (weak -0.80 to -0.92, hypercharge -0.28 to -0.36). Three caveats govern this check. First, on the low end's own input errors alone (without the review's ±0.00004 theory error on the share), the weak row's error narrows to 0.00011, putting the 2025 pure fit's weak residual (-0.000342 at three loops, -0.000358 after four loops) at -3.0 errors at three loops and -3.1 after four loops (-2.8 and -2.9 with the 0.00005 scale band); what currently limits the weak row is the 0.00064 spread between the pure and hybrid third-order Fermi relations. Second, the four-direction conversion $-C_A/(48\pi^2)$ is separated from plain $\overline{\rm MS}$ by 9.8 errors on the weak row (plain $\overline{\rm MS}$ sits at -9.8; the row asks conversion strength 1.02 ± 0.10, selecting whole

strength 1), with a second-order conversion band of 0.11 to 0.22 errors from the unfixed evanescent coupling [51]. Third, the closure fixes a sharp hadronic window bounded by the hypercharge row (colour and weak remain within one error across it): all three rows sit inside one error for $0.027812 \le \Delta\alpha^{(5)}_{\rm had}(M_Z^2) \le 0.027920$ (asking 0.027866) on Martin–Robertson 2019 and $0.027792 \le \Delta\alpha^{(5)}_{\rm had}(M_Z^2) \le 0.027901$ (asking 0.027847) on the 2025 fit (inside two errors for 0.027757–0.027974 and 0.027738–0.027955). Every dispersive e^+e^- determination in Table 10.1 of the 2024 review (0.02755 to 0.02761) is strained outside two errors on the hypercharge row, by −2.0 to −2.5 on its own uncertainty combined with the non-hadronic carry error (−4.4 to −5.8 measured against the fixed baseline 2024 row error before substituting each dispersive evaluation's larger uncertainty), confirming the hadronic fork of section 6 independently at first order.

The running contains no gravitational term. In a mass-independent scheme such as this one, Newton's constant can enter the running of a dimensionless coupling only multiplied by a squared mass of the theory ($Gm_t^2 \approx 2\times10^{-34}$) or by the cosmological term [25]; over the thirty-nine e-folds from the Z to the cell that moves light by less than 10^{-30}. With a momentum cutoff in a particular gauge, gravity's power-law correction would make light's stiffness 1.41 times larger at the cell [23]; it depends on the gauge [24], it is absent in dimensional regularization [25], it does not act as a universal running coupling [26], and in an asymptotically safe completion it would add about 0.18 [27]. Those effects are the size of the higher-dimension terms at the cell. They belong to the matching (section 2.5), which compares the coefficients of the dimension-four term, not to the carry.

The carry assumes that no field carrying electric, weak or colour charge exists between the Z and the cell other than the Standard Model's. That is a premise, and with the closure it becomes a prediction. Every such field softens light at the cell with the same sign. One charged fermion at 1 TeV would move light by about 1,070 of its errors, and one anywhere below 0.966 of the cell's energy (7.08×10^{18} GeV) would move it by more than one error. In the value at rest, the fermion at 1 TeV adds 7.764, giving 144.79 and 144.78. Heavy neutral fields count too when they bind strongly to the lepton doublets: within one error, the sum over them of $y^2\ln(E/M)$ stays below about 7.

The value at rest follows from the value at the cell through one slope. The fine-structure constant enters the carry only through the coupling at the Z, $1/\alpha(M_Z) = (1-\Delta\alpha)/\alpha$. Every vacuum term in $\Delta\alpha$ is itself proportional to α, so $\Delta\alpha/\alpha$ does not move with α, and a change in $1/\alpha$ moves $1/\alpha(M_Z)$ one for one. At fixed α the hadronic input moves $1/\alpha(M_Z)$ by $-1/\alpha$ per unit and light at the cell by −10.88823 per unit. So

$$\frac{d(1/e^2)_{\rm cell}}{d(1/\alpha)} = \frac{10.88823}{137.036} = 0.0794553,$$

close to the one-loop value $1/(4\pi) = 0.0795775$. Solving the carry in full instead of through this slope moves the two boxed values of α^{-1} in section 4 by about -1×10^{-6} and -3×10^{-6}; neither they nor any value of α^{-1} in the table of the carrier's reads that follows them or in Table 4 changes in a printed digit.

4 The fine-structure constant

With $K = 1.040396$, light at the cell is $8K = 8.323168$, against 8.324077 ± 0.000574 and 8.324493 ± 0.000577 asked with the matching's computed second order and the line for what its count leaves open (section 2.5). Those errors already include the dressing's own; that is -1.58 and -2.30 of the error. Through the slope,

$$\boxed{\alpha^{-1} = 137.0246 \pm 0.0072} \quad \text{or} \quad \boxed{\alpha^{-1} = 137.0193 \pm 0.0073}$$

against the measured 137.035999177(21). No continuous number in it was fitted to light: K is computed, the count is whole, the cell is fixed on gravity, and the carry is the Standard Model's own. With the earlier computation of the dressing, $K = 1.040399$, it is 137.0249 and 137.0196 (-1.54 and -2.25). If the 2026 lattice determination's total moved by the 0.00007 our recomputation finds, the result would move up by 0.0096, to 137.0341 and 137.0289, and light would stand 0.26 and 0.98 of its error below its ask; the paper keeps the value as printed.

When every input of the carry that the ledger has its own number for is taken from the ledger instead of from measurement (the strong coupling solved so that colour at the cell is exactly $4K$, which gives $\alpha_s(M_Z) = 0.11835$ and 0.11851, and the quark masses of the ledger's mass table, with the top quark at 172.46 ± 0.37 GeV), light asks 8.323970 ± 0.000542 and 8.324386 ± 0.000545 at the cell and

$$\alpha^{-1} = 137.0259 \pm 0.0068 \quad \text{or} \quad 137.0207 \pm 0.0069,$$

-1.48 and -2.23 of the error. The ledger's values move the result toward the measurement. Of those values, the strong coupling, solved from colour at the cell, rests on the second open premise, that colour's links sit near no turn at the cell (section 2.4).

			first statement		second statement		
read of the carrier	K	earlier	light	α^{-1}	light	α^{-1}	light's read?
long wave, all orders	1.040396	1.040399	-1.58	137.0246	-2.30	137.0193	yes
second order, own covariance	1.040384		-1.75	137.0234	-2.46	137.0181	approximation
second order, Gaussian	1.040377	1.040378	-1.85	137.0226	-2.56	137.0174	approximation
the carrier's own stiffness	1.040759	1.040747	$+3.46$	137.0612	$+2.73$	137.0560	no (§2.4)
mean loop weight	1.0408495	1.040851	$+4.74$	137.0702	$+4.00$	137.0650	no (§2.4)

4.1 What the agreement weighs

The result rests on discrete choices, and they can be counted. Ten were written down, each with its natural alternatives and equal weights, before any combination beyond the eighteen on record had been scored; the reported combination's agreement was already known when they were chosen: light's count (8, 2, 4, 16); colour's (4, 2, 8, 16); which stiffness of the carrier light reads (three); how a triangle weighs its links (five); the weight's coefficient (1, $\frac{1}{2}$, 2); the bonds per event (12, 4, 8); the matching constant (none, minimal subtraction, and three lattice-type constants); the reading scale (six multiples of $\hbar c/\lambda$); the form of the weak share (seven); and the dressing the share reads (three). That is 1,360,800 combinations, each scored on the carry of section 3. None lands light, colour and the weak share within one error. The reported combination lands all three within two under the first statement (-1.58, -0.40,

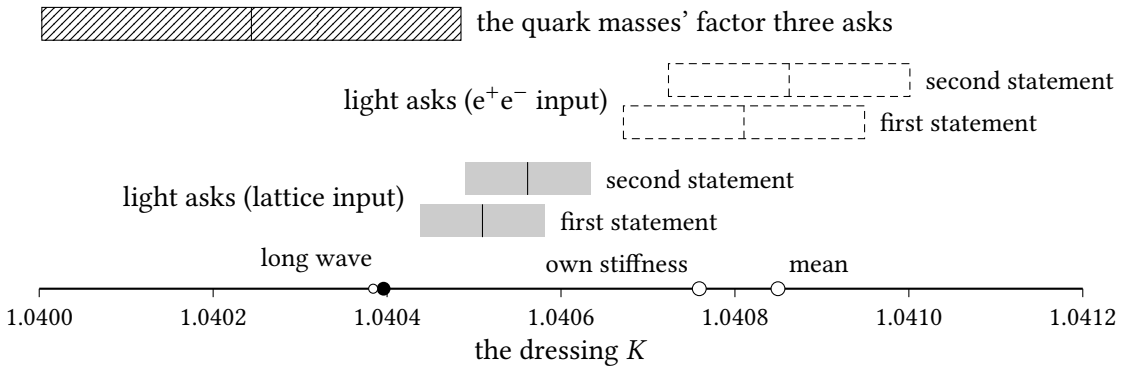

Figure 1. The dressing each measurement asks for, against the carrier's computed reads. Filled dot: the long-wave value, light's read; small circle: its second-order approximation; open circles: the carrier's own stiffness and its mean, which are not light's (section 2.4). Each of light's asks is drawn under both statements of section 2.5, the first statement's below. Light's ask on the lattice hadronic input (grey) lies 1.58 and 2.30 of its error from the long-wave value. The quark masses' factor of three (hatched) asks a dressing within one error of it. On the data-driven hadronic input (dashed) light's ask moves up by 0.0003 under both: there the own stiffness and the mean would land and the long-wave value would not, but neither is a read light can take.

−0.18); under the second none lands all three within two, and the reported combination is the closest of all (−2.30, −0.57, −0.63).

The search for where the logarithms end was wider than any of these menus. Thirty-two forms for the end were tried while it was sought: seven cycle lengths and three walks on the bonds, four ways to weigh a step by its interval, and eighteen closed forms found by hand; more than half put the end between 0.96 and 1.05 of $\hbar c/\lambda$. Before any of them the record already held values for the cost of a posting, none of them derived, some from an earlier attempt on the same constant. Read as one entry's cost, one is the half adopted here: one posting where a debit posting with a credit posting costs one. A cost of $\ln\phi$ puts the end at $0.981\,\hbar c/\lambda$ ($\chi^2 = 11.3$ and 17.0 for three, against 4.7 and 11.9). The record's own convention, one post at strain one, puts it at $2\sinh\frac{1}{2} = 1.0422$, 5.4 and 5.7 errors from the three couplings. One value the couplings cannot tell from the half: $J(9) - J(8) = 71/144$, what one more posting adds to the cost of a count of eight; it is the half less $1/144$ ($\chi^2 = 5.6$ and 12.3). Every other value found there misses by more than ten errors. The earlier attempt has the number one half too, as the least cost of a whole-number flow of the field a posting sources: one flux quantum on the bond that joins the posting's debit to its credit. That is half the square of one unit, as the adopted half is, but in that attempt's own units, and there it is a field's cost. The attempt keeps the cost of the posting itself as a separate number, with candidates of its own. The unit's number does not come from that half. Three of the thirty-two fit the couplings better than the reported package under the first statement and four under the second, all of them walks on the bonds: under the first, cycle of 4 postings (+1.49, −0.25, −0.38); walk on the 24 bonds, in pairs (+1.49, −0.25, −0.38); under the second, cycle of 4 postings (+0.76, −0.42, −0.84); walk on the 24 bonds, in pairs (+0.76, −0.42, −0.84); walk of 16 postings, in pairs (+0.85, −0.25, +0.11); and cycles of three postings, which end the spinors' logarithms at the cell and the vectors' 0.3 per cent below it, put light, colour and the share at −1.10, −0.31 and 0.13 under the first statement and −1.81, −0.49 and −0.33 under the second. It is excluded without using the couplings, twice over: three single postings never close, since an

odd number of half-integer bonds cannot sum to zero (a closed walk of three steps is a triangle, which takes a unit bond, that is two postings); and it is a walk on the bonds, which gravity's two bounds refuse (section 2.5). Twelve of the thirty-two go that way, with no coupling used: two set by the exact cost of a length, which a theorem excludes, and the ten walks on the bonds. Of the twenty left, the couplings keep the end at the cell.

What the agreement weighs is reported here as bookkeeping of the combinations tried, not as a test of significance. Each chance below is the fraction of an assumed range of the couplings' values in which some combination on a menu fits them at least as well as the reported package does (or meets a different bar, where the text names it); it measures how much of that range the menu's combinations cover, and so depends on how many combinations the menu holds. Let the three couplings lie anywhere within one per cent of their values, uniformly and independently, and ask for the chance that some combination fits them at least as well as the reported one does, by the sum of the squared pulls; with the correlated chi-square as the measure instead, the chances given below would be 0.30, 0.037 and 0.000042 under the first statement (given as 0.23, 0.025 and 0.00003) and 0.47, 0.089 and 0.00017 under the second (given as 0.35, 0.055 and 0.00008). Table 2 gives it for each menu, without and with the thirty-two forms for the end; the record's own convention for one post, $\cosh 1 - 1$, is counted in both.

Three numbers say what the agreement weighs, each without and with the search for the end. On the menus fixed before the count, the chance that some combination fits the couplings at least as well as the reported package does is 0.013 and 0.030, and 0.025 and 0.055 with the search (0.0036 and 0.0094 on the ten-choice menu). The one-per-cent range is much wider than the spread of what the candidates predict, and that width keeps these chances small. Conditioned instead on the coarse structure, with each coupling anywhere among the values the candidates of the earlier menu predict for it, some candidate fits all three as well as the reported one with chance 0.12 and 0.21, and 0.23 and 0.35 with the search (light and colour together: 0.62 and 0.92 under the first statement, 0.76 and 0.95 under the second; light alone: 0.98 and 1.00 under the first, 1.00 and 1.00 under the second). After the derivations that were found once the agreement was known, the first chance falls from 0.00071 to 0.00003 under the first statement and from 0.0022 to 0.00008 under the second as each is granted, and from 0.0014 to 0.00003 under the first statement and from 0.0043 to 0.00008 under the second with the twenty forms of the search that survive the exclusions made without the couplings; the last value is for the package with only the weight's coefficient, one or two, left to choose. Each derivation rests on premises that do not mention a coupling (sections 2.2 to 2.5), but each was found with the agreement in view, so we give all of these numbers and do not choose between them; a derivation made without the couplings in view is the test of that. For the counts it is settled: an entry is one comparison of its two sides and is priced once, so the counts follow, and the other pricings miss, two events by thousands of errors; the measured couplings over K, 8.0009 and 4.0019 under the first statement, 8.0013 and 4.0027 under the second, confirm it. It was first found with them in view. These chances take light's matching at the cell at first order with the second-order part computed in section 2.5, and carry what the count leaves open, which of two equal stretches is the line, as a line in each coupling's error. Without that line they would be nearly the same (0.23, 0.025 and 0.00003 under the first statement and 0.35, 0.055 and 0.00008 under the second). They are larger than version 1.7's (0.21, 0.021 and 0.00002) because the computed second order moves light's ask away from the package, and

menu	first statement without	first statement with	second statement without	second statement with
fixed before the count, with the reported agreement already known				
the ten choices above	0.0036		0.0094	
the alternatives each derivation of section 2 was tested against: the counts, the ends of the logarithms, the reads and weightings of the carrier	0.013	0.025	0.030	0.055
the same, with the hadronic input also a choice	0.030	0.055	0.061	0.098
wider: counts up to 24, nine scales, forms of the share off the photon line, the hadronic input a choice	0.052		0.11	
after the exclusions derived once the agreement was known				
the read, the weighting, the matching constant, the ratio of the counts, the bonds per event, the time bond's dressing (and with it the dressing the share reads) and the ends of the logarithms that break a theorem, derived	0.00071	0.0014	0.0022	0.0043
also: a weight acts on odds	0.00057	0.0011	0.0018	0.0036
also: the dressing sits on loops, so the share's form is fixed	0.00014	0.00027	0.00048	0.00092
also: an entry is priced once, as one comparison of its two sides (the number of senses), and a posting that leaves no record is in the least state (the end of the logarithms)	0.00003	0.00003	0.00008	0.00008

Table 2. The chance that some combination of a menu fits light, colour and the weak share at least as well as the reported package, with the three couplings spread uniformly and independently within one per cent of their values, under each of the two statements; without and with the thirty-two forms tried for the end of the logarithms. Each entry is the fraction of that range covered by the menu's combinations, so it depends on how many combinations the menu holds; it is not a test of significance. The upper rows were fixed before the count and count every form tried; the lower rows remove alternatives by derivations found after the agreement was known, and count the twenty forms that no exclusion without the couplings removes. Light's matching at the cell is taken with the second order computed except which of two equal stretches of the links' own quanta is the line; its part is carried in each coupling's error.

the package's worse fit sets a looser bar for every combination, the package itself included.

The precision of the agreement carries little beyond the coarse structure and colour. Given that light's coupling at the cell lies among the carrier's stiffnesses times eight at all, some candidate on the menu fits it as well as the reported one does, almost wherever among those values it lies. What carries the weight is that light and colour sit at whole counts times one stiffness near 1.04 at the cell fixed by gravity, together. Colour's place in the coarse structure rests on the second open premise, that colour's links sit near no turn at the cell (section 2.4).

On every menu the couplings themselves select the counts 8 and 4, the coefficient one, the twelve bonds per event and the reading at $\hbar c/\lambda$, and they leave no weight on any lattice-type matching constant. On the ten-choice menu they lean to pure logarithms (0.87 and 0.98 of the weight); under the first statement to the long-wave value that the derivation takes (0.84) over the carrier's own stiffness (0.04), and under the second to the own stiffness (0.62) over the long-wave value (0.35); the derivations take the long-wave value (sections 2.4 and 2.5). With the derived chain's other choices fixed, the couplings prefer the long-wave read under both statements (0.99 and 0.72 of the weight, the carrier's own stiffness 0.01 and 0.27); the second's lean on the whole menu comes only from counting the own stiffness once for each of the five ways a triangle weighs its links, which move the long-wave read alone. They cannot tell the share's forms apart; that is left to the derivation of the stay (section 5). Of the eleven ends of the logarithms in the derivations' menu, two break a theorem (section 2.5): they are set by the exact cost of a length, and the nearest rival, at $1.0305\,\hbar c/\lambda$, is one of them; that cost weighs a displacement by no law of comparison. Apart from the end at the cell, the other eight each break a reading or a measurement and miss light by more than four errors under the first statement and five under the second.

When the hadronic input is counted as a choice and the read is left free, another rival fits as well. On the electron–positron input 0.02760(10), the carrier's own stiffness or its mean, with the share's form $(2K + 1)/3$, lands all three couplings within one error. Neither part of it survives what is now derived: the own stiffness and the mean are not reads light can take (section 2.4), and the form $(2K + 1)/3$ leaves the time bond undressed, which the lattice's symmetry forbids (section 5). With those exclusions in place the couplings put between 0.81 and 0.88 of the weight on the lattice input under the first statement and between 0.74 and 0.94 under the second, and 1.00 and 0.99 once the ends of the logarithms that break a theorem are removed as well.

These chances take the coarse agreement as given: that light's and colour's couplings at the cell lie within one per cent of whole counts times one stiffness near 1.04 at all. If both couplings at the cell could have been anywhere between 1 and 64 on a logarithmic scale, that alone happens about once in 1,200 with the carrier's own reads at the cell, and about once in 20 with every choice of the ten-choice menu left free. Taking both levels together, the chance that some combination of that menu lands all three couplings within one error is about one in a million (1.1×10^{-6}).

While the dressing was being sought, 120 other values for it were tried: ratios of the ledger's fixed counts, laws of its draw counts, and a few closed forms; ten of them lie between 1.035 and 1.047. Added as further reads to the ten-choice menu with the coefficient and the bonds held, they raise its chance from 0.0017 to 0.011 under the first statement and from 0.0049 to 0.030 under the second; under the first no combination using them fits as well as the reported one, and under the second one of them, 1.04115, fits better than the reported package (+0.84, −1.24, −0.38), with no matching constant and the share's form $D = K$, two choices the derivations remove.

The package was fixed before an input fault in the carry's starting state, found for another reason, was corrected, and the correction moved light toward the measurement with nothing chosen again.

5 Colour, the weak share and the quark masses

Colour. With the same K, colour at the cell is $4K = 4.161584$ against 4.163552 ± 0.004976 and 4.164426 ± 0.004977 asked with the matching's second order: -0.40 and -0.57. Equivalently, the strong coupling at the Z that makes colour exactly $4K$ at the cell is $\alpha_s(M_Z) = 0.11835 \pm 0.00003$ and 0.11851 ± 0.00004 in the adopted convention, the error being the line for what the second order's count leaves open (minimal subtraction at the cell would move it by about 0.001), against the measured 0.1180 ± 0.0009 [15]. Colour's landing and the strong coupling solved from colour at the cell rest on the second open premise, that colour's links sit near no turn at the cell (section 2.4). Colour's window for K is loose (± 0.0012); it confirms that light and colour read one common dressing more than it tests the dressing's value.

The weak share. Light's stiffness splits into the weak part and the hypercharge part, and the split comes from a count on the record of what reads a link. It reads a link's stiffness as the sum, over every carrier that moves with the link, of that carrier's index, the sum of the squared charges of its states. Matter's three generations enter at a weight w: each generation adds 2 to colour, 2 to the weak link, $10/3$ to hypercharge and $16/3$ to light. The links' own carriers enter at weight one: colour's add 3, the weak link's 2, hypercharge's none, and the charged weak carriers add 2 to light. The scalar doublet enters at a weight w_h, adding $\frac{1}{2}$ to the weak link and to hypercharge and 1 to light. So colour counts $6w + 3$, the weak link $6w + 2 + w_h/2$, hypercharge $10w + w_h/2$, and light $16w + 2 + w_h$, the weak and hypercharge counts together. This count fixes ratios only; light's $8K$ sets the size. Light is twice colour, as the loop law requires at every dressing, exactly when $4w + w_h = 4$. Matter is read at its moving share at balance, $w = 1 - \sigma$, where the stay σ is the share of a record's moves that rest; the line then gives $w_h = 4\sigma$, which we call D. The weak count is $8 - D$ and light's is $18 - 3D$, so

$$\frac{1/g^2}{1/e^2} = \frac{8 - D}{18 - 3D},$$

which is $7/15$ when nothing is dressed ($\sigma = \frac{1}{4}$).

The stay is read from the carrier. A recognition has sixteen moves, one for each pair of an account debited and an account credited: four are rests and twelve are transfers. A rest closes one of light's triangles (its two postings and the time bond); a transfer closes none [18]. The lattice's symmetries carry the time bond onto a posting bond, so the rest's triangle is dressed by the carrier's mean loop weight K_m. Light's turn along an open path can be removed by relabelling the phases at its events, so the carrier has nothing on a transfer to dress. If weights act on odds and a move reads the carrier only through the light it closes, the stay is $K_m/(K_m + 3)$ and $D = 4K_m/(K_m + 3)$, which gives the share 0.4674803 against 0.4674908 ± 0.0000584 and 0.4675174 ± 0.0000585 asked with the matching's second order: -0.18 and -0.63. The reading that weights act on odds was adopted with the share in view. Had a move instead weighed the carrier's own cost on the two links it crosses, the share would move by less than a twentieth of its error. The other stay forms on record each break one of these steps: a quarter of the rest's weight does not act on odds, and a triangle with its time bond undressed is ruled out by the symmetry. Writing $D - 1 = r\,(K_m - 1)$, the stay $K_m/(K_m + 3)$ gives $r = 3/(K_m + 3)$, three quarters when nothing is dressed, because twelve of the sixteen moves are transfers. The measured share allows r from 0.70 to 0.80 under the first

statement and from 0.73 to 0.82 under the second, so on its own it would also admit 4/5; no count of the moves gives 4/5.

The weak share's count and its stay are readings, not derivations. The stay rests on two readings, that weights act on odds and that a move reads the carrier only through the light it closes; two of its steps are derived, the time bond's dressing from the lattice's symmetries and, given the second reading, a transfer's weight of one from light's relabelling freedom. The stay's form was adopted with the measured share in view, so it is a located form that lands, not an independent prediction. The share hardly depends on the dressing (its slope is 0.02 per unit of K_m). The weak coupling and hypercharge at the cell are light and the share recombined, not new equations.

The rule that gives light $8K$ and colour $4K$, $1/g^2 = 2STK$ for a link read on carriers of index T, does not give the weak share. Applied to the weak link and to hypercharge separately, each read on the scalar doublet, it gives $4K$ each and a share of one half; the measured 0.4675 excludes that by about 556 and 555 errors. The two counts are one. Light's total and colour's do not depend on which unit charge reads a loop; the division of light between the weak link and hypercharge does. It is the readers' mean split number p: the weak link has $(1-p)/2$ of light's stiffness and hypercharge $(1+p)/2$ (p is the cosine of twice the weak mixing angle at the cell), and p is the mean, over what reads the loop, of each reader's own split number, weighted by its share of light's stiffness. The weak link's own carriers read -1; a body that holds one of the two columns reads 0, which is the count rule's reader and gives the share one half; matter as a whole, all five modes read alike, reads $\frac{1}{4}$; the both-columns record, a debit and a credit on one account, reads 1. The occupation count above is this mean over its census, $(12 \times \frac{1}{4} - 2 + 0)/15 = 1/15$ undressed: the rule's one half assumed that only single columns read. Within a single cost of the two columns' link, $a\ \mathrm{tr}\, X^2 + b\,(\mathrm{tr}\, X)^2$ for the link's generator X, light's stiffness is $a+b$ and $p = b/(a+b)$; the measured share asks $p = 0.0650 \pm 0.0001$, a cost on the link's overall turn worth 6.5 per cent of light's. The sign settles what the pair's credit side is: a cost that compared a body's debit column with its own credit column would raise the share to one half or above, 556 and 555 errors off, so the credit side is the rest of the record (section 2.2). The value of p is not derived, and the weak share is not an independent prediction of this paper: three located forms land it, each found with the measured share in view. They are the occupation count's line with the stay, above, and two given below.

The term that tells hypercharge from the weak link is already in what the ledger reads of a ratio. A complex ratio z has a debit reading z and a credit reading $1/z$, and their mean is $(z + 1/z)/2$. On a bond of size u carrying a turn φ the mean is

$$\cosh(u + i\varphi) = \cosh u\, \cos\varphi \;+\; i\, \sinh u\, \sin\varphi.$$

Light's and colour's stiffness read the real part (section 2.4). The imaginary part is odd in the turn, and it is present only as far as the bond's size unbalances the two readings: it is $\tanh u$ times $\cosh u\, \sin\varphi$, and it vanishes on a bond at balance. On the two columns, turned by φ_1 and φ_2, the imaginary parts add to

$$\sinh u\,(\sin\varphi_1 + \sin\varphi_2) = 2\, \sinh u\, \sin\frac{\varphi_1+\varphi_2}{2}\, \cos\frac{\varphi_1-\varphi_2}{2}.$$

This reads the columns' total turn, which is hypercharge's. It is zero on the weak turn, where the two columns turn oppositely, whatever the size of the turn, and zero on a colour turn that

turns two partners oppositely and leaves the third. So this term tells hypercharge's turn from the weak link's. These are identities of the mean of the two readings; no measurement enters them.

Its size on the carrier is computed. The mean of $\tanh^2 u$ over a bond is $t = 0.0695885 \pm 0.0000011$, from the same sampling that gives the mean loop weight of section 2.3. One form built on t lands the share. Let the columns' total turn cost t where each column's own turn costs one, $a\,(\varphi_1^2 + \varphi_2^2) + a\,t\,(\varphi_1 + \varphi_2)^2$, the two sharing light's $8K$ between them, as weights that act on odds do. Then $b/a = t$ and $p = t/(1+t) = 0.0650610$, hypercharge's stiffness stands to the weak link's as $1 + 2t$ to one, and the share is

$$\frac{1}{2\,(1+t)} = 0.4674695$$

against 0.4674908 ± 0.0000584 and 0.4675174 ± 0.0000585 asked: -0.36 and -0.82. In this form the share's departure from one half is the carrier's doing: with every bond at balance, $t = 0$ and the share is the rule's one half.

The form is located, not derived: it was found with the measured share in view. Two steps are missing. One is why a record that holds both columns should weigh the carrier's mean squared imbalance, one number on every triangle, where a record that holds one column weighs one. The other is why the weights act on odds here; were the term simply added, light would be about 1,000 of its errors high, or colour 55 low. Taken as p itself, t misses by 39 errors. Eighty-eight values have been set against the share, sixty-five of them built on the carrier's bond; fifteen fall within one error of it under the first statement and fourteen under the second, twelve under both. Three of the twelve have a reading behind them: the occupation count's line with the stay (-0.18 and -0.63); $p = K/16$, an account's own rest, one move in sixteen, read as its both-columns record and dressed by K (-0.05 and -0.51); and this one (-0.36 and -0.82). The other nine are one variant of $K/16$, one of the occupation count's line under a move the record does not write, a form tried in earlier work, and six coincidences that were noticed and dropped; five more fall within one error under one statement only: two variants of $K/16$, two estimates made by hand, and one coincidence. The three forms differ by a third of the share's error, so the share does not choose among them. They differ in what they say of a ledger with no carrier: $7/15$, $15/32$ and one half.

Each of the three needs at least one statement the record does not now make, and none is derived. Table 3 sets the three side by side. The share this paper prints is the occupation count's. That form is the adopted one because it was the first of the three to be written, not because the share or free fall chooses it over the other two: versions 1.0 and 1.1 of this paper had its stay and its share alone, a form found with the measured share in view, and the other two entered at version 1.2. The carrier's imbalance as odds needs the columns' total turn to cost one number on every triangle, the carrier's mean t: a weight in a count, as the mean loop weight is for the stay, and not the cost of a bond's own turn. If each triangle instead weighed $\tanh^2 u$ of its own bonds, hypercharge's stiffness would be a relaxed one, as light's is (section 2.4): its links settle into the softer triangles and the total turn's cost falls from t by 0.00177, the weak link's weights are the same on every triangle and its stiffness does not fall, and p comes out 0.06351, 12.9 and 12.4 errors low (with the dressing kept inside each triangle's weights, 9.7 and 10.2 high). The record's own law, on a record holding both columns, gives two lawful totals instead, 100 and 41 errors out at long waves. $K/16$ needs an

account's own rest weighed against the plain sixteen moves, a reading the record once held and then replaced by the normalized stay, on which the own rest's share is 5.5 and 5.0 errors out. The occupation count's line needs matter read at its stay's value at balance, not at its place. The carrier's imbalance uses no stay, so under it the stay's form enters no coupling. The occupation count's line uses the stay at balance, and there, of two forms of the stay, the rests' share lands (-0.18 and -0.63) and a quarter of the rest's weight misses by 4.7 and 4.2 errors. This form holds while a transfer, which closes no loop, weighs one; were the carrier read inside each posting with the pair's half ratio, transfers would be dressed, this form would be 10.5 and 10.9 errors out, $K/16$ would be 16 and 15 out, and of the three forms only the carrier's imbalance, which weighs no move, would still land.

form	split number, and what it is computed from	share	errors, under the first	errors, under the second	whether adopted, and the statement it needs
the occupation count's line with the stay	$p = (2-D)/(18-3D) = 0.0650393$ with $D = 4K_m/(K_m+3)$; $K_m = 1.0408495$, the carrier's mean loop weight	0.4674803	-0.18	-0.63	located, and adopted as the first written; it needs matter read at its stay's value at balance, not at its place
an account's own rest, one move in sixteen, dressed by K	$p = K/16 = 0.0650248$; $K = 1.040396$, the long-wave stiffness	0.4674876	-0.05	-0.51	located, not adopted; it needs an account's own rest weighed against the plain sixteen moves
the carrier's imbalance as odds	$p = t/(1+t) = 0.0650610$; $t = \langle \tanh^2 u \rangle = 0.0695885$, the carrier's mean squared imbalance	0.4674695	-0.36	-0.82	located, not adopted; it needs the columns' total turn to cost one number on every triangle, the carrier's mean t, as a weight in a count

Table 3. The three located forms that land the weak share; none is an independent prediction of it. A split number p gives the share $(1-p)/2$; the fourth and fifth columns are the form's share less the share asked, in errors of the share, under the first statement of section 2.5 (0.4674908 ± 0.0000584 asked) and under the second (0.4675174 ± 0.0000585). None of the three is derived: each was found with the measured share in view, and each needs at least one statement that the record does not now make. They differ by a third of the share's error, so the share does not choose among them. With no carrier they give the shares $7/15$, $15/32$ and one half.

A combinatorial check on the three-loop carry from the three two-loop low-end determinations of section 3 (2024 inputs) sharpens what the share asks of the occupation count's line. Writing $D = 1+3q$, so that the weak share is $(7-3q)/(15-9q)$ with q the chance that a credit answers its own debit, those three determinations ask q about once in 89 to 101. The ninety-six triangles at an event plus the pair's own debit ($q = 1/97$, share $338/723 = 0.4674965$) puts colour, weak and hypercharge within one error on both pure $\overline{\text{MS}}$ determinations ($-0.42, +0.08, -0.87$ on Martin–Robertson 2019 and $-0.42, -0.68, -0.37$ on Martin 2022) and sits outside one error on the hybrid determination's weak row (-1.92), whereas the power-of-two candidate $q = 2^{-7}$

($893/1911 = 0.4672946$) misses all three on the weak row by -3.72 to -5.71 errors and cannot arise from a balanced four-account cell with whole counts on each account, whose own-answer denominator $4k + 1$ is always odd (code archive, `four_loop_three_coupling/`). Because ninety-six was located with the weak share in view and the one-error window is six to nine counts wide, this is a consistency check on the same datum, not a derivation or a new closure.

Free fall: the split's number is not a local stay. A form whose number follows matter's stay at its own place would make the weak share depend on gravity. Near a still body the stay rises with the depth of the potential, $\sigma = \sigma_0 + (1 - \sigma_0)B$ with $B = 2|U|/c^2$, so matter's moving share falls as $(1 - \sigma_0)(1 - B)$. Read there, the occupation count's line moves the weak share at the cell by $+0.080\,B$. In ordinary matter the proton's weak charge, $1 - 4\sin^2\theta$, then changes, and with it each nucleus's weak neutral-current energy: about 1.4 eV a nucleon in titanium and 1.7 in platinum (zero-range exchange in a uniform nucleus). Titanium and platinum would fall differently by $\eta = 3.2 \times 10^{-11}$, or a tenth of that if correlations between nucleons suppress the energy tenfold. MICROSCOPE measured $\eta(\mathrm{Ti}, \mathrm{Pt}) = (-1.5 \pm 2.3 \pm 1.5) \times 10^{-15}$ [53], so the line read this way is out by more than a thousand of its errors. The same measurement holds any response of the weak share to the potential below about $2 \times 10^{-5}\,B$ at two errors ($2 \times 10^{-4}\,B$ with the suppression). A split that follows the local stay is therefore out unless its response is several hundred times weaker than the line's.

The forms from the carrier's imbalance and from $K/16$ read constants of the carrier instead, which would move with light's dressing, and clocks bound the dressing's response to the potential, as below. The carrier's imbalance holds no stay, and it moves with light's dressing: both are set by the spread of the bonds' sizes, and a shift of the carrier moves the imbalance about 1.5 times as fast as the dressing. The dressing in $K/16$ is the one light reads, and its constancy is measured: if it followed the Sun's potential, so would α, and clock comparisons bound that at $(c^2/\alpha)\,d\alpha/d\Phi = (14 \pm 11) \times 10^{-9}$ [54], which keeps the weak share's response under $10^{-9}\,B$. For the carrier's imbalance the same bound keeps it under $2 \times 10^{-8}\,B$. The occupation count's line stands if matter is read at its stay's value at balance rather than at its place. The ledger does not make that choice, and the printed share needs it. So free fall shows that the split's number is not a local stay; it does not choose among the three.

One sentence would cover the counts and the division together: *two columns held in two records are compared; two columns held in one record read their mean.* The first half is the pair reading of section 2.2, a body against the rest of the record, and gives the counts 8 and 4. The second half is the record that holds a debit with its credit on one account: in the mean of its two readings a turn's first order cancels except through the imbalance, which is the term above. This is a working statement, assembled with all three couplings in view. It is not derived.

Because the share hardly depends on the dressing, it asks for the hadronic input by itself: 0.027807 ± 0.000061 and 0.027771 ± 0.000062, 1.6 and 2.7 combined errors from the value light asks. The two errors share part of the carry, so this is a consistency, not two independent data: two of the package's equations take the same side of the disagreement between the hadronic determinations, and one of them, the weak share's, hardly depends on the dressing. The share's own ask lies 0.18 and 0.63 errors below the 2026 lattice value and 1.8 and 1.5 above the data-driven 0.02760(10).

The quark masses. On the ledger's mass table [58], three times each quark's number at the cell, counted in golden steps above the electron, sits on a whole place plus a small term, and that factor of three depends on colour's stiffness at the cell. Run at the long-wave value, the six quark rows give a factor of 2.9970 ± 0.0047, 0.63 of its error below three. The dressing at which the factor is exactly three, found by running the table, is 1.040244 ± 0.000241 (figure 1); the long-wave value lies +0.63 from it, the carrier's own stiffness +2.1 and the mean +2.5. The six terms of that table were fitted to the quark masses, so this is a check on fitted terms, not on fitted light; it is the one other measured equation that asks for the value of K, and the computed K lies between the two asks.

Two numbers it was never compared with. The same package predicts two quantities that played no part in choosing anything in it. With the fine-structure constant, the Fermi constant and the top and Higgs masses as measured, and the hadronic input, the strong coupling and the Z mass solved so that light, colour and the weak share hold at the cell, it gives the W mass as 80.3512 ± 0.0048 GeV and the leptonic effective weak mixing angle as $\sin^2\theta_{\text{eff}} = 0.23160 \pm 0.00005$. Both were computed, with the rule for scoring them written down, before they were compared. The 2026 averages are 80.3625 ± 0.0077 GeV, 1.25 combined errors above, and 0.23150 ± 0.00013, 0.68 below [16]; both pass. Through the strong coupling, the W mass and the angle rest on the second open premise, that colour's links sit near no turn at the cell (section 2.4). With the matching's second order, computed in the centre, and with the error carrying the open choice of which of two equal stretches is the line, the same route gives 80.3512 ± 0.0048 GeV and 0.23160 ± 0.00005 under the first statement and 80.3527 ± 0.0048 GeV and 0.23157 ± 0.00005 under the second, 1.25 and 0.68, and 1.08 and 0.46 combined errors from the averages. The Standard Model alone, on the same inputs, sits at 1.37 and 0.86 errors, so the W mass and the angle barely tell the package from the Standard Model. They do test that the dressing enters the stay: with the rest undressed, the angle would miss by 4.0 and 4.2 errors. Against the 2023 combination of all W-mass measurements, 80.3946 ± 0.0115 GeV, which includes the CDF measurement [38] and which its authors find disfavoured [37], the package misses by 3.5 and 3.4, and the Standard Model by 3.6. The Z mass the package solves for, with the Fermi constant setting the scale and the matching's second order included, is 91.1954 and 91.2014 GeV. The measured value is 91.1876 ± 0.0021 GeV, so the package lies 7.8 and 13.8 MeV above it. The carry's own error is 5.0 MeV and the open choice of which of two equal stretches is the line adds 0.6 and 1.0 MeV, so that is 1.42 and 2.48 combined errors. This is the 1.6- and 2.7-error gap between the hadronic inputs the weak share and light ask for, seen in the Z mass. It is not an independent test.

6 Where it does not close

The closure's error is the carry's, and the largest part of the carry's error is the hadronic input $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$: how much the quark loops screen light between rest and the Z. It cannot be computed in perturbation theory at low energy. It is determined two ways, from the rate at which electrons and positrons annihilate into hadrons and from lattice computation, and the two disagree. The disagreement is the same one that divides the predictions of the muon's magnetic moment: the 2025 consensus evaluation of that moment set the electron–positron data aside for its hadronic part until the conflicts among the data sets are understood [17].

		first statement		second statement	
determination	$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$	light	α^{-1}	light	α^{-1}
lattice, Conigli et al. 2026 [39]	0.027821(49)	−1.58	137.025(7)	−2.30	137.019(7)
lattice (Mainz 2022), Erler et al. 2024 [43]	0.02794(9)	+0.39	137.041(13)	−0.03	137.036(13)
lattice (BMW), Erler and Ferro-Hernández 2023 [42]	0.027842(63)	−0.95	137.027(9)	−1.52	137.022(9)
lattice, Mainz 2022 [40]	0.02773(15)	−1.15	137.012(21)	−1.41	137.007(21)
lattice (BMW 2017), de Blas et al. 2022 [41]	0.02766(10)	−2.40	137.002(14)	−2.77	136.997(14)
world average, 2024 review [15]	0.02783(6)	−1.18	137.026(9)	−1.78	137.021(9)
e^+e^- and 2023 pions, Di Luzio et al. 2024 [44]	0.02775(12)	−1.27	137.015(17)	−1.58	137.010(17)
e^+e^- and 2023 pions, Narison 2025 [46]	0.027707(45)	−4.03	137.009(7)	−4.78	137.004(7)
e^+e^- and 2023 pions, Narison 2023 [45]	0.027663(45)	−4.92	137.003(7)	−5.68	136.998(7)
e^+e^- data to 2019 below 2 GeV, Erler et al. 2024 [43]	0.02762(5)	−5.30	136.997(7)	−5.99	136.992(7)
e^+e^- data to 2019, Davier et al., input of the 2026 fit [16]	0.02760(10)	−2.99	136.994(14)	−3.36	136.989(14)
e^+e^-, Erler and Ferro-Hernández 2017, in the review's table [16]	0.02761(10)	−2.89	136.996(14)	−3.26	136.990(14)
e^+e^-, de Trocóniz and Ynduráin, in the review's table [16]	0.02754(10)	−3.58	136.986(14)	−3.95	136.981(14)
lattice $\Pi(0)$ at $m_\pi = 185$ MeV with perturbation theory, Dominguez and Hernández 2023, five flavours [52]	0.027486(73)	−5.54	136.979(10)	−6.03	136.973(10)
electroweak fit, input left free [16]	0.02734(33)	−1.71	136.959(45)	−1.82	136.953(45)

Table 4. Determinations of the hadronic input, light's pull at the cell on each (in standard errors, the carry's and the dressing's together), and the fine-structure constant the closure gives with each, under each of the two statements.

Table 4 lists the determinations and what the closure gives on each.

On the determinations that rest on a lattice computation at the physical pion mass the closure holds within 1.6 errors on four of the five under the first statement, one of which lies above the value it asks, and the fifth, the one that takes its low-energy part from the lattice data of 2017, lies 2.4 below; under the second it holds within 1.6 on three of the five, one of which lies on the value it asks, and the other two, the 2026 determination and the one from the lattice data of 2017, lie 2.3 and 2.8 below. These determinations differ as much in how they carry the lattice data to the Z as in the data themselves: the Mainz data of 2022 give 0.02773 carried with the Adler function from $Q^2 = 5$ GeV^2, and 0.02794 when only the light quarks are taken from the lattice, at $Q^2 = 4$ GeV^2, and the charm and bottom quarks and all the running above are taken from perturbation theory (an earlier calculation of the same kind gave 0.02795(11), +0.41 and +0.07). On the 2024 world average and on one electron–positron evaluation with the 2023 pion data the closure holds as well. The six electron–positron determinations in Table 4 with errors

of 0.0001 or less lie between 2.9 and 5.3 errors below under the first statement and between 3.3 and 6.0 under the second, and a determination that takes from a lattice computation only the light quarks' correlator at zero momentum, at a pion mass of 185 MeV, and everything else from perturbation theory lies 5.5 and 6.0 below. A fuller census of thirty-three determinations and fit values, from 2001 to 2026, shows the same pattern.

The 2026 lattice determination [39] adds to its lattice part the running from $Q^2 = 9\ \mathrm{GeV}^2$ to M_Z^2, computed from the Adler function in perturbation theory and printed as 0.019304(35) (table 7 of the second paper cited). We recomputed this piece with the code the determination names, AdlerPy 0.1.3 [55], as the integral it states, with the inputs it cites from the 2024 lattice averages [56]: $\alpha_s(M_Z) = 0.1183$, $m_c(3\ \mathrm{GeV}) = 0.991\ \mathrm{GeV}$ and $m_b(m_b) = 4.171\ \mathrm{GeV}$. The installation reproduces the code's own published example to its last printed digit. We varied, one at a time, each setting the determination does not state: four or five loops in the massless terms, the electromagnetic correction on or off, two choices of the charm and bottom threshold settings, and the gluon and quark condensates on or off. Every one of these settings reproduces the printed steps of the same table from 5 to 12 GeV^2 to within 0.9×10^{-6}, and gives the piece up to M_Z as 0.019371 to 0.019379, which is 0.000067 to 0.000075 above the printed value. (Pole masses for charm and bottom do not reproduce the printed steps and are set aside.) Ending the integral at $Q = 90.0\ \mathrm{GeV}$ instead of M_Z gives 0.019294 to 0.019302, close to the printed value but 2.5 to 10.4×10^{-6} below it, outside the table's own agreement, so that end is not the cause as it stands. The cause is not known. If the determination's total moved by 0.00007, it would be about 0.02789(5): on the value this closure filed in advance, 0.027882 ± 0.000019, and less than half an error below the value it asks with the second order as computed under the first statement, 0.027905 ± 0.000020, and 1.0 combined errors below the value asked under the second, 0.027943 ± 0.000020 (section 7). Table 4 and the rest of this paper use the value as printed. The scripts that give every number in this paragraph are in the code archive.

The weak share, which hardly depends on the dressing, says the same in the adopted convention: it lands at -0.18 and -0.63 on the lattice input and at $+1.76$ and $+1.46$ on the data-driven input of the 2026 fit. The two rows share the convention, the scale and the carry, so this is one convention's verdict, not two independent ones; in minimal subtraction the same form of the share would side the other way. The read does not rescue the closure on the electron–positron side: the reads that would land there are not light's (section 2.4). The closure's verdict is the lattice's against the data's.

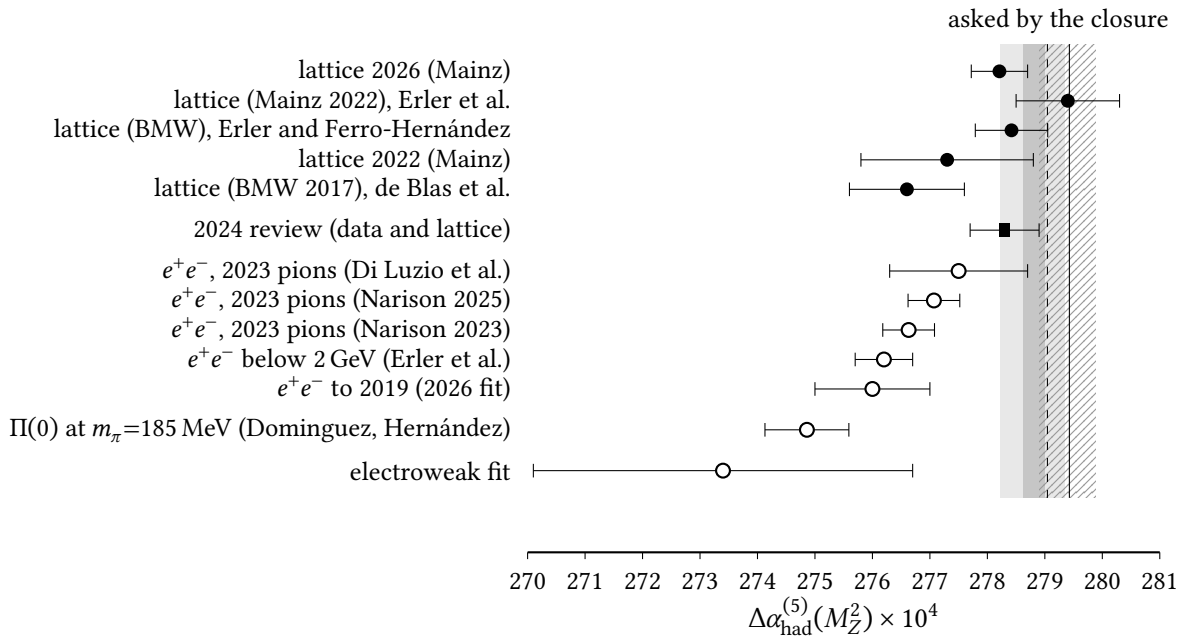

Figure 2. The hadronic input as determined (one standard error) against the values the closure asks: the hatched band is the value filed with version 1.8, 0.027939 ± 0.000048, and the two lines the values asked under the two statements (dashed: the first statement; solid: the second); the dark band is the value filed with version 1.1, 0.027882 ± 0.000019; the light band is the value filed with version 1.0, 0.027878 ± 0.000056. Filled circles: lattice computation at the physical pion mass; square: the 2024 world average; open circles: electron–positron data, a lattice value at a heavier pion mass with perturbation theory, and the electroweak fit. The filled circles lie within 2.4 and 2.8 combined errors of the two values asked, one of them above the first and on the second; the electron–positron values with errors of 0.0001 or less lie 3.0 to 5.3 and 3.4 to 6.0 below.

7 The test

A determination of the hadronic input outside the filed windows, scored by the filed rules, falsifies the closure; that is what filing it means.

The hadronic input, as filed with version 1.0. With α measured, the closure fixes the hadronic input it needs, the value that makes light land exactly. Version 1.0 filed it, on 14 November 2026, as

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027878 \pm 0.000056$$

(0.0278785 ± 0.0000561 before rounding), the error being the carry's other errors over the slope 10.888, with this rule: a new determination with an error of 0.00010 or less that lies within two combined errors of it passes; one that lies beyond three fails it, and with it the closure as stated, unless the carry itself is shown to be wrong; between two and three, it strains it. By that rule the closure fails on a new determination of error 0.00010 below 0.027534, or of error 0.00005 below 0.027653. (Computed from the printed 0.027878 ± 0.000056, the second boundary is 0.0276528, so a value between it and 0.027653 strains rather than fails.) That filing stands with its numbers.

The hadronic input on the corrected carry. The same rule on the corrected carry, with the dressing of section 2.3, is a second filing, also dated 14 November 2026:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027882 \pm 0.000019,$$

with the measured strong coupling and top mass, or 0.027872 ± 0.000008 with the ledger's own. By it the closure fails on a new determination of error 0.00010 below 0.0275761, or of error 0.00005 below 0.0277209 (with the ledger's inputs, below 0.0275706 and 0.0277196). Both filings score only determinations first made public after 14 November 2026: those published before were in view when the rule was written, and they lie on both sides of the value asked (section 6). The test is of the package as stated, with the long-wave read. Changing the read after a determination would be a new hypothesis, not a rescue, and the read is not free (section 2.4). With the carry's three-loop completion taken from a shared start (section 3) the closure asks 0.027880 ± 0.000019 (0.027870 with the ledger's inputs); the difference is 0.07 of the filed error, and the filing stands as filed. With the matching's computed second order and the line for what its count leaves open (section 2.5) the closure asks 0.027905 ± 0.000020 and 0.027943 ± 0.000020 (0.027895 ± 0.000009 and 0.027933 ± 0.000010 with the ledger's inputs). The filing 0.027882 ± 0.000019 stays as filed: it is scored by its own rule against every new determination, and the value the package asks at the time is printed beside it. With version 1.8 a new value was filed, dated and hashed, with its own rule below, and both filings are kept. No determination made public since 14 November 2026 has scored either of the earlier two.

The hadronic input with the second order computed. The same rule on version 1.8's chain is a third filing, dated 15 November 2026:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027939 \pm 0.000048,$$

with the measured strong coupling and top mass and the line for the choices the record does not state in its error, or 0.027929 ± 0.000045 with the ledger's own. By it the closure fails on a new determination of error 0.00010 below 0.0276054 or above 0.0282722, or of error 0.00005 below 0.0277298 or above 0.0281478 (with the ledger's inputs, below 0.0275995 or above 0.0282583, and below 0.0277264 or above 0.0281314).

The strong coupling. The same package gives the strong coupling at the Z as 0.11836 ± 0.00003 in its convention at first order, thirty times sharper than its present measurement; with the matching's computed second order it is 0.11835 ± 0.00003 and 0.11851 ± 0.00004 (section 2.5), a shift inside the rule's allowance. The rule for it was filed on the carry of version 1.0, where the package gives 0.11835: a new world average or determination $y \pm \sigma_y$ (minimal subtraction, five flavours, $\sigma_y \le 0.0005$) is scored by $t = |y - 0.11835| / \sqrt{\sigma_y^2 + 0.00003^2 + 0.0003^2}$, where 0.0003 is an allowance, fixed with the rule, for the carry's truncation and its matching at 200 GeV in this number: $t \le 2$ passes, $t > 3$ fails. For $\sigma_y = 0.0005$ the package fails below 0.1165984 or above 0.1201016.

The W mass and the weak angle. A later world average of the W mass with an error of 0.005 GeV fails the package as stated if it lies below 80.3304 or above 80.3720 GeV under the

first statement and below 80.3319 or above 80.3735 GeV under the second, and a later average of the effective angle with an error of 0.0001 if it lies below 0.231254 or above 0.231937 under the first statement and below 0.231224 or above 0.231906 under the second. The Standard Model's own values on the same inputs sit close to the package's, so such a failure would first be the Standard Model's. Neither replaces the hadronic test.

A second measured equation: the muon. The quark loops that screen light between rest and the Z also make the largest hadronic contribution to the muon's anomalous magnetic moment, $a_\mu^{\mathrm{HVP,LO}}$. Both are integrals of the same function, the vacuum polarization of the colour-charged quarks, weighted differently: the muon reads 93 per cent of it below $Q^2 = 0.25\ \mathrm{GeV}^2$, the running of α reads it up to the Z. The closure's asked input therefore implies a value of $a_\mu^{\mathrm{HVP,LO}}$, and the measured muon tests it. The rule was fixed before computing: the comparison is with what the measured anomaly asks of $a_\mu^{\mathrm{HVP,LO}}$ once every other contribution of the 2025 Standard Model evaluation is subtracted, and a pull beyond two standard errors strains the closure, beyond three fails it.

The measured anomaly, $a_\mu = 116\,592\,071.5(14.5) \times 10^{-11}$ [47], less the other contributions of the 2025 evaluation [48], asks $a_\mu^{\mathrm{HVP,LO}} = (717.05 \pm 1.76) \times 10^{-10}$. To turn an asked input into a value for the muon we need the shape of the vacuum polarization below $Q_0^2 = 9\ \mathrm{GeV}^2$. Above it the running is perturbative (0.019304 as the lattice determination prints it, see section 6, plus 0.000045 from the space-like to the time-like Z), so the value filed with version 1.1, 0.027882 ± 0.000019, fixes the part below it: 0.008533, which is 0.72 per cent above the lattice value 0.008472. We take the shape from the same lattice group, which also gives $a_\mu^{\mathrm{HVP,LO}} = 724.5(7.1) \times 10^{-10}$ [50], and scale the muon's value by the same 0.72 per cent. The two lattice quantities are strongly correlated (a correlation of 0.8 has been published between a lattice muon value and its running at the Z [43]), so their ratio is sharper than the muon's value alone: its error is 0.71 per cent against 0.99. The filing then implies

$$a_\mu^{\mathrm{HVP,LO}} = (729.7 \pm 6.2) \times 10^{-10},$$

1.97 standard errors above what the measured muon asks: inside the rule, at its edge. With the correlation set to one the error is 5.5 and the pull 2.21, a strain. With the ledger's own strong coupling and quark masses in the carry the filed input is 0.027872 and the pull 1.77 (2.00 at full correlation). On the value filed with version 1.0 the pull is 1.57. Against the 2025 lattice average, $713.2(6.1) \times 10^{-10}$ [48], it is 1.90. On the value the closure asks with this version's second order, 0.027905 ± 0.000020 and 0.027943 ± 0.000020, the muon's part is $(731.6 \pm 6.2) \times 10^{-10}$ and $(734.9 \pm 6.2) \times 10^{-10}$, 2.26 and 2.75 standard errors above what the measured muon asks: a strain; with the correlation set to one, 2.53 and 3.09. With the perturbative piece above $9\ \mathrm{GeV}^2$ as we recompute it (section 6), 0.019371 to 0.019379 instead of the printed 0.019304, the pull is 1.27 to 1.38 and 1.77 to 1.88, and it passes.

The lattice's shape matters as much as the number. Read the same way, every asked input between 0.02759 and 0.02788 would pass at the error filed with version 1.1 and at this version's, so the muon does not by itself choose between the determinations of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$. What separates them is the shape: the 2020 evaluation of $a_\mu^{\mathrm{HVP,LO}}$ from electron–positron data, $693.1(4.0) \times 10^{-10}$ [49], lies 5.5 standard errors below what the measured muon asks, and the lattice evaluations lie within about one.

On the value filed with version 1.1 the closure asks 0.000061 more of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ than the lattice determination of 2026, and the muon reads only the part of that excess that sits below 9 GeV^2. With full correlation the muon passes if no more than 0.000047 of it sits there; at least 0.000014, nearly a quarter of it, must then sit in the perturbative running above 9 GeV^2, where the muon does not look. If the whole excess sat there, the muon's pull would be +1.2 (+1.3 with full correlation): what remains is the lattice's own muon value, which lies above the measured muon's ask. On the value asked with this version's second order the excess is 0.000084 and 0.000122; the muon passes with full correlation if no more than 0.000047 and 0.000048 of it sits below 9 GeV^2, so at least 0.000036 and 0.000074, 43 and 61 per cent of it, must sit in the running above. Our recomputation of that running (section 6) gives 0.000067 to 0.000075 more than the printed value.

8 The chain, step by step

The table lists every step from the ledger to light's stiffness at the cell, and the carry from there to rest, with its status. *Derived*: follows from the steps above it and the premises named, and is conditional on those premises where they are readings. *Reading*: an identification adopted and built on, not yet derived; what would derive it is named. *Selected*: what the step is, with every alternative counted and missed by the measured couplings; they were in view when it was first found, so they select the step and do not test it. *Checked by measurement*: the same, with the alternatives missed by a measurement other than the couplings. *Borrowed*: standard physics, used as it stands. *Located, not adopted*: a form found with the measured share in view that lands the share, where another form is the adopted one.

step	status	what it rests on, or what would derive it
the cell's edge $\lambda = 2\sqrt{\ln 2}\,\ell_P$	reading	one bit of record per λ^2 of area, fixed on gravity, not on light
the one bit a face counts gravity's whole stiffness, the fields' closed histories included	checked by measurement*	under the count of the matching the known fields' closed histories supply 1.57 per cent of that stiffness; if the bit counted the record's own stiffness only, the cell would be 0.78 per cent larger and light would stand at -2.23 and -2.94; counted and missed through the ledger's law for the electron's mass in units of gravity, which has no free number: it holds at 0.81 of G's errors with the whole stiffness and misses by 689 with the record's own stiffness only; *that law is a working one, fixed on one datum, and measured G alone reads neither
the lattice of events, 24 bonds	reading	the ledger's postings, four accounts with two columns; given it, the 12 bonds per event and the triangles are counted
the cost $J = \cosh\ell - 1$	derived	uniqueness of the reciprocal cost [20]

step	status	what it rests on, or what would derive it
the cost acts as a weight (coefficient one)	selected	not a continuous setting; counted and missed: the cost in the amplitude, a coefficient of two (light 281 errors low), and a coefficient of one half (light 548 and 546 errors high); light locates it at 0.9972 ± 0.0018 and 0.9959 ± 0.0018, light's shortfall (section 9); derived if the record's weights are shown to be its costs
each bond's fluctuation, $\langle \ell \sinh \ell \rangle = 1/12$	derived	the identity on the weight (section 2.3)
light reads the relaxed long-wave value, not the mean or the carrier's own stiffness	derived	what a coupling is: the coefficient in light's law at long waves, its links obeying their law in the carrier's weights; that energy splits into the relaxed cost and a part that does not depend on the loop value
light reads the carrier without weighing it	reading	the same reading as "the links' own turns are not summed as weights at the cell"; summed with the carrier, light's turns would move its coefficient to about $35/24$ and put colour ten errors low under the first statement and eleven under the second
the triangle weighs its links' mean	reading	each link reads a loop's whole turn (parallel); given it, the lattice's symmetry and the balance value force the mean; shared in series the harmonic mean, 22 and 23 errors low
$K = 1.040396 \pm 0.000002$	derived	computed; an earlier computation gives 1.040399 ± 0.000005
$1/g^2 = 2STK$ on the 24 bonds	derived	the lattice's triangles and their second moment
the ratio of light's count to colour's, two	derived	the ratio of their generator traces, given that light turns one of a body's two columns and colour the three partners; the rule gives light's and colour's totals only
the pair reading: an entry is one comparison of its two sides, priced once	selected	what an entry is: its debit read against its credit, the cost of z^2, sets $S = 4$ and leaves K unchanged; priced as two events, the sum applied inside one entry, light is 7,247 and 7,219 errors low (the record's own sums add postings, which is the unit), as nothing it has no stiffness, read on one side or with three senses thousands of errors off; under the record's law of composition, a premise, one record of the two sides costs nothing or the cost of z^2, never the sum; measured $S = 4.0004 \pm 0.0003$ and 4.0006 ± 0.0003 (light) and 4.002 ± 0.005 and 4.003 ± 0.005 (colour); the credit side is the whole rest because nothing less balances the entry; first found with the couplings in view; it also gives the matching's unit (with an entry's two sides priced apart the end would miss by 49 errors)

step	status	what it rests on, or what would derive it
the imaginary part of the mean of the two readings reads the columns' total turn only	derived	the mean of a complex ratio's two readings, $\cosh(u + i\varphi) = \cosh u \cos\varphi + i \sinh u \sin\varphi$: on the two columns the imaginary parts read the total turn and cancel on the weak turn; the term is $\tanh u$ of the real one and vanishes at balance
a record holds each link's unseen part	checked by measurement	counted and missed: a uniform hidden weight would stop unit charges from propagating, and free charges exist; the quarks' thirds need it in any form
no matching constant	derived	the summed count (section 2.5)
the links' own turns are not summed as weights at the cell	reading	chosen with the couplings in view; a lawful weight sees no turn; summed, the turns would add constants to the matching that measurement excludes, and would make light weigh the carrier (colour ten errors low under the first statement, eleven under the second); summed under the pair alone, colour's turns on the 24 bonds would sit at the half turn (a triangle's mean reading -0.19, against $+0.89$ under four times the one-sided cost); not summed, the pair alone leaves them on the half-turn side too (-0.27 at K), so this reading alone does not carry colour from the cell
colour's links sit near no turn at the cell	reading	built on by colour's carry from the cell, by the strong coupling solved from colour at the cell, and by the W mass and the mixing angle predicted with that coupling; section 2.4 proves a sharp no-go boundary at $S = 4$ (ruling out single-triangle pair costs, carrier dressing, five-mode neutrality, and pure-axis/pure-diagonal even loops including pure-axis eight-tick cycles, and bounding quarks 96–270× short when speed-matched, 55.4× at raw unit speed) and classifies all exact-$S = 4$ single-triangle exits (quartic coefficient $a < 9$) alongside a 528-loop triangle-commutator pure-pair exit ($\eta_c = 0.00866/0.00949$ at $L = 4$, finite-coupling Monte Carlo open)

step	status	what it rests on, or what would derive it
a posting that leaves no record is in the least state of the equal-weight action, in entries and radians	selected	sets the matching's unit; the principle is in the ledger's posting law, and the number one half is on the record beforehand as the cost of one posting where a debit posting with a credit posting costs one; counted and missed by the end the three couplings ask: the record's own convention, strain one (5.4 and 5.7 errors), the other values it orders, one golden rung, one bit and one coin a posting (45 or more), an entry's two sides priced apart (49), the turn in whole cycles (121), the square of the least state (46), and every other value found on the record (more than eight); a cost of $\ln \phi$ stands 2.6 and 2.3 errors off; $71/144$ is counted and not missed (1.0 and 0.7 errors); one half is the ledger's own value; the radian, the bond and the least state were chosen with the end in view; measured: one entry 0.515 ± 0.011 and 0.522 ± 0.011 against $\frac{1}{2}$ (the same datum as light's and colour's landings)
the inside of a history that leaves no record is not on the record's places	reading	lattice places inside such a history are refused by gravity's wave speed and the graviton's mass (no coupling enters); with it every walk on the bonds is excluded, the one form that fits the couplings better among them
the unrecorded count is summed	selected	the count of a history that leaves no record is not on the record, so it is summed, every whole number of units weighing one over itself; that gives the matching no constant at first order and one end for every spin; counted and missed: the lattice read as a regulator (light thousands of errors low), minimal subtraction at the cell (light more than eight errors low under the first statement, nine under the second), and a cut on each line of a loop (40 and 38 errors from the one combination of the couplings that does not involve the end, which stands 2.2 and 3.4 from zero); identified with the couplings in view; to be derived from "nothing outside the record" on weighted histories
at second order, an exchanged link meets a history as the link that is read does	reading	the one working reading the second order's count adds to the first order's counting (section 2.5); with it a link's stretch inside a body's history is counted as a line of the link's own quantum, the only complete count of the three possible in form; for the top's closed history with the scalar inside the scalar's stretch is counted as an exchanged link's is, and the scalar's mass, like the bodies', is held at the pole of its dressed line; the record does not state it

step	status	what it rests on, or what would derive it
where the links' own quanta meet one another, the longest stretch is the line, summed plainly, and the others close the loop	reading	the record does not state this count as a rule; it meets the two conditions the record fixes, no mass for the links' own quanta and the published two-loop running, exactly; which of two equal stretches is the line is open and carried in the error; derived if the record said that, where a history of one field's quanta has two cycles, the loop counted at one over its count is the shortest cycle and the remaining stretch is the line
where a stretch of the loop grows as long as the line, its moved term is counted there once (kept in the loop) or summed over the line's whole counts (carried into the line)	reading	open: the record makes neither statement (section 2.5); both give no mass and the published running exactly, and both values remain reported; within continuum foliation only the second statement ($\gamma = \infty$, 137.0193) removes both artificial boundary jumps (86.9% of the gap is the first statement's far-end cusp at line-to-loop ratio $b = 1$) and finite discrete line-telescoping schemes cluster around it (+0.338 to +0.433), while fixing δ_5 unconditionally requires a native discrete closure-preserving two-cycle kernel on the 24-bond lattice
gravity's strain adds nothing at the cell	selected	a strain is the record's own geometry, sourced and never summed; counted and missed: the strain's own histories summed like a field's, which in a cutoff scheme would raise light by a factor 1.41, where two errors leave room for a relative addition of no more than 0.0003 under either statement; identified with the couplings in view; to be derived from the strain's own law
the carry from the cell to rest	borrowed	the Standard Model's running, three loops with computed shifts; the ledger has the two ends, not yet the passage
no other charged body below the cell	reading	the mass table's bodies; tested by every search for new charged or coloured states
the time bond is dressed like a posting bond, so a rest weighs K_m	derived	the lattice's symmetries
a move that closes no loop keeps weight one	derived	light's relabelling freedom, given that a move reads the carrier only through the light it closes (a reading that moves the share by 0.03 of its error)
a weight acts on odds	reading	adopted with the share in view; derived if the record's move law is shown to normalize over the sixteen moves
the stay $K_m/(K_m + 3)$, $r = 3/(K_m + 3)$	reading	from the three rows above, and conditional on them; r, defined by $D - 1 = r\,(K_m - 1)$, is three quarters when nothing is dressed; adopted with the measured share in view

step	status	what it rests on, or what would derive it
the weak share's count: each carrier's index, matter at its moving share, the scalar's weight by the line	reading	adopted with the measured share in view; the division of light is the readers' mean split number p (the rule's one half assumes single-column readers only); the value of p, 0.0650 measured, is not derived; three forms land it
the columns' total turn costs one number on every triangle, the mean squared imbalance $t = \langle \tanh^2 u \rangle = 0.0695885$, as odds: $p = t/(1+t)$	located, not adopted	located with the share in view and not adopted in place of the occupation count: share 0.4674695, −0.36 and −0.82; one of 88 values tried, 65 of them from the carrier; three forms land (−0.18, −0.05, −0.36; −0.63, −0.51, −0.82) and the share does not choose; a cost bond by bond does not give it (p comes out 12.9 and 12.4 errors low at long waves); derived if a record holding both columns is shown to weigh the carrier's mean squared imbalance against a single column's one, the weights totalling one
two columns held in two records are compared; two columns held in one record read their mean	reading	a working statement that would give the counts and the division of light together, assembled with all three couplings in view; its first half is the pair reading; derived if the ledger's cost is shown to compare records, with the credit side the whole rest of the record

A step that is a reading is called one wherever it appears in this paper. The readings were identified with the couplings in view; section 4.1 counts what that costs. Read on one side, the same structure gives $2K$ for light and K for colour, which measurement excludes, and summing the links' own turns at the cell would give the matching constants that measurement excludes: the counts are what an entry is, selected by the couplings, and the matching rests on steps that are selected, checked by measurement or read, not on the structure alone.

8.1 The premises in one list

Every row of the chain table whose step is not derived from the rows before it is listed here once, in the table's order, with its tag: *proved*, derived from a theorem proved in a cited work, under that theorem's hypotheses; *checked*, settled against measurement, either selected among counted alternatives by the measured couplings, which were in view when it was found and so select it and do not test it, or checked by a measurement other than the couplings (each item says which); *working*, a reading adopted and built on, not yet derived; *borrowed*, standard physics used as it stands; *located, not adopted*, a form found with the measured share in view that lands the share, where another form is the adopted one. Each is a premise of the results except the one row tagged *located, not adopted.* Two rows' notes qualify their tags, and where a note and its tag differ, the note is exact: the checked row on a posting that leaves no record is selected by the couplings, but one other counted value, 71/144, is not missed by them either; and the working row on a stretch of the loop as long as the line adopts neither statement, since the record makes neither, and carries both, which give the two values. The derived steps follow from these premises and from the conditions named in their own rows, and are

conditional on them; the table above says for each what it rests on, or what would derive it.

working the cell's edge $\lambda = 2\sqrt{\ln 2}\,\ell_P$

checked the one bit a face counts gravity's whole stiffness, the fields' closed histories included; checked by measurement, through a working law fixed on one datum

working the lattice of events, 24 bonds

proved the cost $J = \cosh \ell - 1$; the uniqueness of the reciprocal cost, cited

checked the cost acts as a weight (coefficient one); selected by the couplings

working light reads the carrier without weighing it; the same reading as the links' own turns not summed, below

working the triangle weighs its links' mean

checked the pair reading: an entry is one comparison of its two sides, priced once; selected by the couplings

checked a record holds each link's unseen part; checked by measurement

working the links' own turns are not summed as weights at the cell

working colour's links sit near no turn at the cell; open: the record does not state it; colour's landing, the strong coupling solved from colour at the cell, and the W mass and the mixing angle rest on it

checked a posting that leaves no record is in the least state of the equal-weight action, in entries and radians; selected by the couplings; one value, $71/144$, is counted and not missed

working the inside of a history that leaves no record is not on the record's places

checked the unrecorded count is summed; selected by the couplings

working at second order, an exchanged link meets a history as the link that is read does

working where the links' own quanta meet one another, the longest stretch is the line, summed plainly, and the others close the loop

working where a stretch of the loop grows as long as the line, its moved term is counted there once (kept in the loop) or summed over the line's whole counts (carried into the line); open: the record makes neither statement; both are carried, and they give the two values

checked gravity's strain adds nothing at the cell; selected by the couplings

borrowed the carry from the cell to rest

working no other charged body below the cell

working a weight acts on odds

working the stay $K_m/(K_m + 3)$, $r = 3/(K_m + 3)$

working the weak share's count: each carrier's index, matter at its moving share, the scalar's weight by the line; the value of p, 0.0650, is measured, not derived

located, not adopted the columns' total turn costs one number on every triangle, the mean squared imbalance $t = \langle \tanh^2 u \rangle = 0.0695885$, as odds: $p = t/(1 + t)$; the adopted form is the occupation count's

working two columns held in two records are compared; two columns held in one record read their mean; this statement's first half is the pair reading listed above, so its working part is the second half, the record that holds a debit with its credit on one account

9 What it means, and what is still owed

The fine-structure constant is light's turn stiffness, read at rest. At the ledger's smallest cell that stiffness is a whole count times one computed number, and the number comes from the ledger's own carrier: twelve bonds to an event, the cost of a comparison at coefficient one, and the stiffness a long wave reads from it. Carried down to rest, it is 1/137.025 or 1/137.019. The magic number is a count and a stiffness.

Several steps of the chain follow from stated premises: the ratio of light's count to colour's, which stiffness light reads, the absence of a constant in the matching once the unrecorded count is summed, and the dressing of a rest. Each is conditional on its premises, and the premises that carry the physics are readings, several of them now selected among counted alternatives, except the counts: an entry is one comparison of its two sides, priced once, and light and colour confirm it. What remains to derive is short and exact: that a posting which leaves no record is in the least state of the equal-weight action, with the radian, the bond and the least state as chosen, which sets the matching's unit; that the cell's one bit a face counts gravity's whole stiffness; that a record holds each link's unseen part; that the count of an unrecorded history is summed; that the links' own turns are not summed as weights at the cell; that colour's links sit near no turn at the cell; that gravity's strain is never summed; that the cost acts as a weight at coefficient one; that each link reads the whole of a loop's turn; and the readings that give the weak share's form, a weight acting on odds among them. The division of light between the weak link and hypercharge is the readers' mean split number, and its value is not yet derived. Light's landing can be read back as the number of senses (4.0004 ± 0.0003 and 4.0006 ± 0.0003), one entry's cost (0.515 ± 0.011 and 0.522 ± 0.011 against $\frac{1}{2}$), the weight's coefficient (0.9972 ± 0.0018 and 0.9959 ± 0.0018) or the postings to a unit (7.76 ± 0.16 and 7.66 ± 0.16); these are one datum re-expressed with the other readings held fixed, not four confirmations. That datum is light's shortfall, 1.58 and 2.30 of its errors: a tension, one number whether it is read as the dressing left free, the coefficient light locates or a common strain. The next hadronic determinations will move it, and a native discrete closure-preserving two-cycle kernel on the 24-bond lattice would settle the finite matching constant unconditionally (section 2.5; within continuum foliation only the second statement, 137.0193, avoids artificial boundary jumps and matches the discrete telescoping cluster, while one unchanged shared-link reader now yields all five two-loop running constants $A = 34/3$, $C_s = 4$, $B_s = 2/3$, $C_b = 2$ and $B = 10/3$ natively). Colour's own landing puts the senses at 4.002 ± 0.005 and 4.003 ± 0.005. The carry from the cell down to rest is the Standard Model's computed running, borrowed whole: the ledger has the two ends, and the passage between them is still the borrowed calculation. Its error is the closure's error, and its hadronic part decides the test filed in section 7.

The pricing that sets light's count also gives the matching's unit its number. Costed on its pair, a comparison presents the square of its ratio. A hop's turn θ presents 2θ, which is the count's four. A posting's golden entry presents ϕ^2, which costs one half. A hop is one entry; a recognition's debit posting and credit posting are two, and they add to one. So two numbers of the closure, light's four and the unit's one half, rest on one pricing where they rested on two readings. The measurements are the ones already given. With the entry's two sides priced apart, the sum applied inside one entry, the count would be two and one entry would cost $2J(\phi) = 0.236$: the logarithms would end at $0.69\,\hbar c/\lambda$, 49 errors from the end the

three couplings ask. Its source is the entry's own structure: one comparison of its two sides (section 2.2).

Appendix: the second order's constants, part by part

This appendix rebuilds the two values from the page. For each part of the matching's second order it gives the weight and its source, the count's sums, the two constants whose half difference is the part's end, and the shift of light's ask; it closes with the sum. Every figure is printed by a file of the code archive, in the folder `second_order_matching` or `links_own_quanta`; the file is given in brackets, with the label of its line where the file labels its lines. Each figure is rounded from its file's full value, so a product or a difference of printed figures can miss the printed result by one unit in its last place.

What the parts have in common. A part is a closed history with a second line inside it, read by the link whose coupling is matched. Its weight is a density $j(b)$ in the inner line's stretch b, in units of the closed history's length, with the two places where the line meets the closed history anywhere along it. Under the count of section 2.5 the closed history is whole at one over its count N and the inner line is whole from one, so the density is summed at $b = n/N$:

$$S[j] \;=\; \sum_{N\geq 1} \frac{1}{N}\Big[\frac{1}{N}\sum_{n\geq 1} j\big(\tfrac{n}{N}\big) - J\Big], \qquad J = \int_0^\infty j\,db.$$

Here J is the integral of the weight itself and $S[j]$ is the constant the count adds to it: the weight's own term is $J\,L + S[j]$, with $L = \ln(1/\lambda^2 m^2)$. A part's logarithm holds more than the weight's own term. The mass m that cuts the long histories is held at the pole of its dressed line under the count and under minimal subtraction alike, so each line that carries the inner line enters with a logarithm and a constant of its own. And in parts 2 and 5 the long end of the inner line's stretch adds to the coefficient. There the weight falls as $J_1(D)\,b^{1-D/2}$ in D dimensions, with $J_1(4) = 0$, so the integral J at four dimensions does not hold it. Summed at $b = n/N$ beyond any fixed $n = M$, for D above four, that tail is $J_1(D)\,N^{D/2-2}\,\zeta(D/2-1,\,M+1)$, and since $\zeta(1+x,\,a) = 1/x + \ldots$ it tends to $2J_1'(4)$ as $D \to 4$, the same at every count N [`corner_under_count`, E3]. So the long end adds $2J_1'(4)$ to the part's coefficient and nothing to its constant. Collected, the part's logarithm is $W\,[L + K]$ under the count and $W\,[\ln(\mu^2/m^2) + K_{\rm MS}]$ under minimal subtraction: W is the part's whole coefficient, called the whole below, and K is the part's whole constant divided by the whole. The part ends $\delta = (K - K_{\rm MS})/2$ e-folds above $\hbar c/\lambda$, or below it where δ is negative; the archive's files print this end as `c`. In this appendix K is always a part's constant, as the archive's files print it; the dressing of section 2.3, which the text also writes K, is named in words. Light's ask moves by the part's slope times δ. The slope is the part's share of the couplings' published two-loop running at the cell, for one e-fold of the end; there $g_2^2 = 0.25670$, $g_Y^2 = 0.22561$, $g_3^2 = 0.23981$, and the top's coupling to the scalar has $y^2 = 0.14619$ [`second_order_slopes`, S0]. Slopes, and the shifts of light's ask, are in units of 10^{-6}, as in the table of section 2.5; the six slopes add up to light's whole second-order slope, -902.0 [S1].

1. Spinor histories with another thing's link inside. A quark's or a lepton's closed history with a colour, a weak or a hypercharge link inside, where the link that is read meets the body only. *Weight:* the published two-loop weight of a spinor's closed history with a vector inside in a constant field, at second order in the field and before the vector's proper time is integrated [57]; $J = 2$, the source's value [`spinor_ring`, R5]. *Count:* $S[j]/J = -2.01927$, by two routes of summation [R6]; the body's own line with the link on it ends at $+\frac{1}{2}$ exactly [R4]. The line enters through the mass that cuts the closed history's first-order logarithm, at twice the whole, and the weight's own term enters with a minus sign: in the weight's units $4\,[L + \frac{1}{2}] - 2\,[L - 2.01927] = 2\,[L + 3.01927]$, so $K = 2 \cdot \frac{1}{2} + 2.01927 = 3.01927$ [R7]. *Minimal subtraction:* the same combination of the source's two dimensional constants, $\frac{4}{3}$ for the line and $-\frac{13}{12}$ for the closed history: $K_{\mathrm{MS}} = 2 \cdot \frac{4}{3} + \frac{13}{12} = \frac{15}{4}$ [R1]. *End:* $\delta = (3.01927 - 3.75)/2 = -0.36536$ [R7]. *Slope, and the shift of light's ask:* $-[\frac{104}{9}\, g_Y^2 + 12\, g_2^2 + \frac{80}{3}\, g_3^2]/128\pi^4 = -969.0$, the sum of this kind's terms in the two-loop running [R8]; light's ask moves by +354.03.

2a. The read link on the exchanged link's own line, on spinor closed histories. The same closed histories where the link that is read meets the exchanged link's own line; only the weak links and colour, which turn their own quanta, have such a part. *Weight:* derived from the action's graphs in any number of dimensions [`weight_d`]; in every dimension its integral is one half of the published two-loop expression [63], the half being a convention of units [`dim_integral`, D2]. The whole is $\frac{10}{3}$ [D3]. In four dimensions the weight at fixed proper times has $J = 1$, three tenths of the whole [`ja_closed`, C2]; the other $\frac{7}{3}$ is the long end of the link's stretch, the $2J_1'(4)$ of the opening, where the closed history is a point on the link's line [`dim_integral`, D4, D5]. *Count:* on the part at fixed proper times $S[j] = -1.49476$ [`piece_r_count`, K3]. The long end adds no constant, as the opening shows, and a direct sum confirms it: as the number of dimensions goes to four, the count's sum over the link's whole stretches exceeds its four-dimensional value by $\frac{7}{3}$ at each of the four counts of the closed history tried, to 6×10^{-11} [`corner_under_count`, E1, E2]. So K is the sum divided by the whole, $(-1.49476)/\frac{10}{3} = -0.44843$. *Minimal subtraction:* $K_{\mathrm{MS}} = -\frac{8}{15}$ [`piece_r_count`, K4]. *End:* $\delta = +0.04245$ [K4]. *Slope, and the shift of light's ask:* $-40\, g_2^2/128\pi^4 = -823.52$; light's ask moves by −34.96.

2b. The same on the scalar's closed history. The scalar's closed history has the same part [`scalar_weight_d`; `scalar_integral`, Q1, Q5]. The whole is $\frac{1}{3}$: the weight at fixed proper times has $J = -\frac{1}{4}$, and the long end gives $\frac{7}{12}$, again $2J_1'(4)$ [`scalar_integral`, Q3]. The count's sum is $S[j] = +0.24256$ [`scalar_count`, C3], so K, the sum divided by the whole, is three times it, +0.72769, the product taken before rounding [C6]; $K_{\mathrm{MS}} = -\frac{13}{12}$ and $\delta = +0.90551$ [C6]. *Slope, and the shift of light's ask:* $-\frac{2}{3}\, g_2^2/128\pi^4 = -13.73$; light's ask moves by −12.43 [C7].

3. Scalar histories with another thing's link inside. The scalar's closed history with a weak or a hypercharge link inside, where the link that is read meets the scalar only. *Weight:* derived from the action's graphs in any number of dimensions [`scalar_weight_d`]; its integral there stands in one fixed ratio, $-\frac{1}{8}$, to the published scalar weight's [57] at each of

the four values of the dimension tried, 3.3, 3.7, 4.3 and 5.5 [`scalar_integral`, Q1, Q2]. In four dimensions a short link gives the weight a power term, the link's power-like addition to the scalar's squared mass, which the scalar's own line cancels at every count [Q6]; with it taken out $J = \frac{3}{2}$ [Q5]. *Count:* $S[j] = -1.88157$ [`scalar_count`, C2]; the scalar's own line with the link on it ends at $+\frac{3}{2}$ exactly [`scalar_integral`, Q6]. The line enters through the mass that cuts the closed history's first-order logarithm: the link adds $3\,[L + \frac{3}{2}]$ to the scalar's squared mass, in that mass's own units and the power term aside, and one sixth of it enters [Q6; `scalar_count`, C5]. Here the weight's own term and the line enter with the same sign: the closed history's $\frac{3}{2}L - 1.88157$ and the line's $\frac{1}{2}\,[L + \frac{3}{2}]$ make $2\,[L - 0.56579]$, so the whole is 2 and $K = -0.56579$ [`scalar_count`, C5]. *Minimal subtraction:* the closed history's $-\frac{2}{3}$ and the line's $\frac{7}{6}$, over the whole: $K_{\mathrm{MS}} = \frac{1}{4}$ [C5]. *End:* $\delta = -0.40789$ [C5]. *Slope, and the shift of light's ask:* $-(g_Y^2 + 3g_2^2)/128\pi^4 = -79.86$ [C7]; light's ask moves by +32.57.

4. The top's closed history with the scalar inside. Section 2.5 gives the derivation; these are its figures. *Weight:* derived from the action, $A(b)\,(q_1^2 + q_2^2) + B(b)\,q_1 q_2$ for stretches with charges q_1 and q_2 under the link that is read [`top_scalar_weight`, W1, W2]. Where a stretch of the quark and the scalar's stretch are both short the weight's integral converges only conditionally [`top_scalar_integrals`, I6], so its value depends on how it is taken. In the count's order, the two meeting places first and the scalar's stretch after, $\int A\,db = \frac{8}{3}$ and $\int B\,db = \frac{4}{3}$ [I4]. With the three lines, $-\frac{4}{3}\,(q_1^2 + q_2^2)$ from the two stretches' own and $+\frac{2}{3}\,(q_1 - q_2)^2$ from the scalar's own, that is the published two-loop running, $2\,(q_1^2 + q_2^2)$, for every pair of charges [31] [I7]. Carried to any number of dimensions the same two integrals are $\frac{14}{3} - \frac{7}{3}\,\epsilon$ and $12\,\epsilon$ at $4 - 2\epsilon$ dimensions [`top_scalar_dimensions`, D3, D4]: at four dimensions $\frac{14}{3}$ and 0, where the count's order has $\frac{8}{3}$ and $\frac{4}{3}$. The conditional convergence allows the difference, and each value goes with its own lines. In any number of dimensions a scalar without mass has no first-order logarithm, so the scalar's own line is absent there, and the weight with the two stretches' own lines alone has the published pole, $2\,(q_1^2 + q_2^2)/2\epsilon$, with no product term [`top_scalar_constant`, K3]. *Count:* $S[A] = -3.85495$ and $S[B] = +4.31619$ [`top_scalar_count`, C1]; each stretch's own line ends at $-\frac{1}{2}$ and the scalar's own line carries no constant [`top_scalar_lines`, L1, L3]. The whole is $2\,(q_1^2 + q_2^2)$, so $K = [S[A] + \frac{4}{3} \cdot \frac{1}{2} + \rho\,S[B]]/2 = -1.59414 + 2.15810\,\rho$, with $\rho = q_1 q_2/(q_1^2 + q_2^2)$ [`top_scalar_constant`, K2]. *Minimal subtraction:* $K_{\mathrm{MS}} = -\frac{5}{4} + 3\rho$ [K3]. *End:* $\delta = -0.17207 - 0.42095\,\rho$ [K4]: -0.38255 where the scalar is neutral to the link that is read ($\rho = \frac{1}{2}$, colour), -0.17207 for the weak links ($\rho = 0$), -0.27112 for hypercharge ($\rho = \frac{4}{17}$), and -0.00369 where light reads the charged scalar ($\rho = -\frac{2}{5}$). Light's row weighs the weak links and hypercharge nine to seventeen, their $q_1^2 + q_2^2$ being $\frac{1}{4}$ and $\frac{17}{36}$, or the neutral and the charged scalar eight to five, $\frac{8}{9}$ and $\frac{5}{9}$ by electric charge. Either way the mean ρ is $\frac{2}{13}$ and $\delta = -0.23683$ [K5], with $K = -1.26213$ [`v192_derived_numbers`, V10] and $K_{\mathrm{MS}} = -\frac{5}{4} + \frac{6}{13} = -\frac{41}{52}$. *Slope, and the shift of light's ask:* $\frac{13}{3}\,y^2/128\pi^4 = +50.81$; light's ask moves by -12.03. *Checks:* a second derivation from the action gives the same weight and the same sums; the one other count of this history gives the quark a mass and does not run as published [`top_scalar_other_counting`].

5. The links' own quanta meeting one another. The closed history of one field's quanta with two cycles: three stretches of the links' own quanta joined at two places, or two closed stretches touching at one. *Weight:* the published two-loop weight of the gauge field in a constant field [61], with its ghosts [62] and the coincident-end and figure-eight terms of [61], expanded to second order in the field [`ssz_expand`; `psz_ghost_contact`]; derived again from the action's graphs in four to eight dimensions and equal term by term [`textbook_S`; `textbook_SD`]. One pole term recomputes as $+\frac{1}{12\epsilon}$, the sign [62] prints and [61]'s own total needs [`ssz_poles`, S3]. *Count:* the count leaves one choice to a statement the record does not make: where a stretch of the loop grows as long as the line, its moved term is counted there once (kept in the loop, the first statement) or summed over the line's whole counts (carried into the line, the second). Both meet the two facts the record does fix: no mass in any number of dimensions [`tail_D3`, D3; `placement_rows_check`, L3], and the whole $-\frac{68}{3}$ [L5]: the published two-loop running $\frac{34}{3}C_A^2 = \frac{136}{3}$ of the weak links ($C_A = 2$), in the weight's units, which carry a factor $-\frac{1}{2}$ (a convention of units). Under the first the weight in the inner line's stretch has $J = -\frac{32}{3}$ and the long line adds -12 [`placement_rows_check`, L5]; $S[j]/J = -0.48959$ [`slices_check`]. Under the second $J = -14$ [`second_statement_count`, S5] and the long line adds $-\frac{26}{3}$ [`placement_rows_check`, L4]; $S[j]/J = 1.06973$ [`second_statement_count`, S6]. K is the sum divided by the whole: $\frac{8}{17}(-0.48959) = -0.23040$ [`slices_check`] and $\frac{21}{34}$ of 1.06973, +0.66071, the product taken before rounding [`second_statement_count`, S7]. *Minimal subtraction:* the same weight in D dimensions with the same cut, $K_{\mathrm{MS}} = -0.094863$ [`J_D`, M1]. *End:* $\delta = -0.0678$ and +0.37779 [`second_statement_count`, S7]; the first, unrounded, is -0.067767 [`slices_check`], and the runs take it at four places. Which of two equal stretches is the line the count leaves open; it is carried in the error, ± 0.048 and ± 0.073 in δ, half the spread between the neutral and the charged stretch as the line [`slices_check`; `second_statement_count`, S7]. *Slope, and the shift of light's ask:* $\frac{136}{3}\, g_2^2/128\pi^4 = +933.33$ [`v193_appendix_numbers`, A5]; light's ask moves by -63.28 under the first statement and by +352.60 under the second [A5].

6. The sum. Light's ask at first order is 8.32381334 (section 2.5; the row "First order" of the table in section 3 prints it to seven places). The six shifts of light's ask, from parts 1, 2a, 2b, 3, 4 and 5, are +354.03, −34.96, −12.43, +32.57, −12.03 and, for the links' own quanta, −63.28 under the first statement or +352.60 under the second [`v192_derived_numbers`, V9]. In all they are +263.90 or +679.78, so light's ask is 8.32407724 or 8.32449312, printed as 8.324077 and 8.324493. With light's value at the cell, eight times the dressing, 8.323168, and the slope 0.0794553 of section 3,

$$\alpha^{-1} = 137.035999 - \frac{8.324077 - 8.323168}{0.0794553} = 137.0246$$

under the first statement, and under the second

$$\alpha^{-1} = 137.035999 - \frac{8.324493 - 8.323168}{0.0794553} = 137.0193.$$

The two runs that carry every part through the three couplings with their covariance print the same asks and the same two values [`v192_sweep_once_count_output.txt`, `v192_sweep_line_sum_output.txt`, X1, X2].

Code and data

The computation of the carrier's three stiffnesses (the mean loop weight, its own stiffness, and light's long-wave stiffness at all orders and at second order), the stored runs, their analysis and the figure of their dependence on the lattice size are public at

```
https://recognitionphysics.org/publications/fine-structure-constant/FineStructureFromTheLedger-code-v2.1.zip
```

and in the paper's Zenodo record, doi:10.5281/zenodo.24227501, which resolves to its newest version. They are written in Python with NumPy; each run checks the lattice, and each run that solves for light its solver, against the definitions before it starts, and the archive lists the command for every run and the checksum of every file. Archive version 1.2 added the computation of the matching's second order (section 2.5), the recomputation of the 2026 lattice determination's perturbative piece and an independent rerun of it (section 6), the menus and chances of what the agreement weighs (section 4.1), the weak share's form against its ask with the bounds from free fall and from clocks, and the carry from the Z to the cell, with its code from the companion paper on the mass table, that gives the W mass, the weak angle and the Z mass (section 5). Archive version 1.3 added the run behind every number on light's and colour's side (the one version 1.8 printed, with the runs for the range of readings beside it), the matching's second order where the read link meets the exchanged link's own line and on the scalar's closed histories, the second order where a field's quanta meet one another, colour's turns on the twenty-four bonds and the hadronic input's table and bounds, and brought the electroweak, free-fall, weak-split and odds computations to version 1.8's numbers. Archive version 1.4 prints the muon's passing ranges rounded inward with their exact edges, and states which value of light's stiffness the runs of colour's turns used, with a check that no printed digit depends on it; it also holds the computations that draw the two figures, and its odds computations take the printed values of the carrier's own stiffness and its mean loop weight. Archive version 1.5 holds the two runs it prints, made by the same sweep, with a file that prints every number the paper reads from them; the electroweak numbers, the stay rows and the weak share's phrases on both statements; the carry table (Table 1) with every number of section 3; the run that finds the dressing at which the quark masses' factor is exactly three; and every number of the hadronic input's subject that no earlier file printed. Archive version 1.6 adds to version 1.5 the sum of the count of the links' own quanta under the second statement, whose constants version 1.5 took as typed inputs, a check that neither statement gives a mass in any dimension, and the three stays set against the share asked behind the reworded phrase on the quarter. Archive version 1.7 adds the computation of the top quark's closed history with the scalar inside and the second order's sweep that carries it link by link, version 1.9.2's two runs, one for each statement, and every folder whose printed numbers or figures move on them, rerun on those runs. Archive version 1.8 adds the file that prints every figure of the appendix; for the carry table, a last column that marks each line of light's error as within one reading set or across readings, and two further printouts, the carry solved in full beside its slope and light's error within one reading set apart from the spread across readings; a file that prints the paper's phrases on the weak share as version 1.9.3 words them, with light undressed set against its ask; a checksum list in each of the two folders on the 2026 lattice determination's perturbative piece; and, at the top, the three requirements files and the one script described below, with its list of commands. Archive version 2.0

updates `colour_turns_24_bonds/README.md` (and its checksum list) to match the colour-ground correction of version 1.9.5, adds `pure_link_two_loop_A/` (running `pure_link_A.py` and `rate_split.py` to compute $A = 34/3$, its mechanism and rate splits, and the four-link meeting-term corner-log cancellation) and `four_loop_three_coupling/` (running `four_loop_check.py` against `beta4_coeffs.json` to verify all 51 lower-order monomials and complete four-loop Standard Model gauge slopes, pure-versus-hybrid low-end pulls, hadronic windows, dispersive pulls and the $1/97$ versus 2^{-7} weak-split check), and registers all three commands in `rerun_all.sh`. Archive version 2.1 extends `pure_link_two_loop_A/` with `hand_engine.py` and `hand_reader.py` (running the unchanged uniform-cut reader on the hopping pair and the Weyl-fermion matter hand to obtain $C_b = 2$ and $B = \frac{10}{3}$, together with exact symbolic checks of the four-link meeting term under sharp cut, power law, smooth profile and cut rescaling) and adds `open_premises_analysis/` (nine standalone deterministic scripts verifying the two-cycle placement family, Euler–Maclaurin cusp decomposition, foliation non-invariance, discrete telescoping and three-stretch mass obstruction, and the SU(N) involution manifold, 24-bond Bloch even-loop dichotomy, quark-sea bounds, global exact-$S = 4$ Steiner-deltoid classification and 528-loop triangle-commutator exit). The one-count two-cycle test ($-\frac{76}{3}$, section 2.5) and the un-summed colour least-state runs ($-0.27, -0.30, -0.32$, section 2.4) remain stated from companion computations outside the archive.

Reproducing. The archive is `FineStructureFromTheLedger-code-v2.1.zip`, sha256 `b87e91bf374391c49894382310b39fb70eb05ba213541edd54e0f3ba22748bdc`. The stored outputs were made with Python 3.10.12, in three environments, each pinned in its own requirements file: the dressing's runs with NumPy 2.2.6 alone (`requirements-dressing.txt`); the two folders of the lattice piece below with NumPy 2.2.6, SciPy 1.15.3, mpmath 1.4.1, AdlerPy 0.1.3 and rundec 0.7 (`requirements-adlerpy.txt`); and every other folder with NumPy 1.21.5, SciPy 1.8.0, SymPy 1.12 and mpmath 1.2.1 (`requirements-main.txt`). One script, `rerun_all.sh`, reruns the dressing's commands and every folder's, in order and each under its environment. After each command it checks that folder's checksums, and at the end every checksum list and that no command left a file the archive does not hold. If an interpreter does not have exactly the pinned versions, the script stops and says what to install. It reruns the long sampling runs, which take hours, only when given `--sampling`. `RUNS.txt` gives the command and seed of every stored run. From those runs one command regenerates the dressing,

```
python3 analyze.py outputs --prefix run --json outputs/results.json
```

whose summary gives $K = 1.0403963 \pm 0.0000016$, printed here as 1.040396 ± 0.000002. The recomputation of the 2026 lattice determination's perturbative piece is in `mainz_upper_end/`: `mainz_upper_end.py`, run for every setting by `run_all.sh` after `check_install.py`, with the inputs the determination states ($\alpha_s(M_Z) = 0.1183$, $m_c(3\text{ GeV}) = 0.991$ GeV and $m_b(m_b) = 4.171$ GeV from the 2024 lattice averages, $M_Z = 91.1876$ GeV) and the code it names, AdlerPy 0.1.3; the independent rerun is in `mainz_rerun/` (`mainz_rerun.py`, settings S1 to S5, and `compare.py`).

About this version

Version 2.1 (26 November 2026) updates version 2.0 of the same day with four structural additions and two minor typography fixes; neither headline value of $1/\alpha$ moves (137.0246 ± 0.0072 and 137.0193±0.0073, −1.58 and −2.30 errors). First, in section 2.3, a spurious $1/N$ factor is removed from the second-order covariance display (with the momentum-space covariance stated explicitly, matching `gauss2.py`) and the Gaussian second-order dressing printed in the text is corrected from 1.040378 to 1.040377 to match the table of the carrier's reads in section 4 and `gauss2_closure.log`. Second, section 2.4 adds the exact SU(N) involution-manifold theorem (explaining why SU(3) is uniquely minimal among SU(N) and Standard Model gauge groups with $d_1 = 4$), the 24-bond three-frame parity and Bloch-spectrum even-loop dichotomy, the Dirac and Wilson quark-sea bounds (96 to 270 times short when speed-matched, 55.4 times at raw unit speed), the global Steiner-deltoid classification of exact-$S = 4$ single-triangle costs ($a \leq 9$ and $a - 3b \leq 8$), and the 528-loop triangle-commutator pure-pair exit with its zero-mode audit ($\eta_c = 0.00866$ harmonic, 0.00949 Euclidean Gaussian at $L = 4$; finite-coupling Monte Carlo open). Third, section 2.5 adds the structural analysis of the two-cycle placement: the one-parameter continuum family $\gamma \in [1, \infty]$, zero-artificial-jump uniqueness of the second statement ($\gamma = \infty$, 137.0193), the 86.9 per cent Euler–Maclaurin cusp breakdown, the exact foliation non-invariance theorem ($+\frac{15}{128}\zeta(3)$), shared-link κ^0 scale invariance at $D = 4$, the three-stretch discrete mass obstruction ($+\frac{40}{3}\zeta(3)$), and the discrete line-telescoping cluster (+0.338 to +0.433 e-folds). Fourth, section 2.5 upgrades the native shared-link derivation of $A = \frac{34}{3}$: all seven face finite parts are established analytically by ray integration (all $\ln 3$ cancelling identically), the four-link meeting term's finite part is distinguished across sharp cut (0), power law (0) and smooth profile (−24, with opposite $+24 \ln s_{\text{cut}}$ and $-24 \ln s_{\text{cut}}$ shifts giving strict cut-scale invariance of A), and running the same unchanged uniform-cut reader on the Weyl-fermion matter hand with D free gives $C_b = 2$ and $B = \frac{10}{3}$ on the first run, closing all five two-loop constants from one reader under the four remaining limits. The code archive is updated to version 2.1, extending `pure_link_two_loop_A/` with `hand_engine.py` and `hand_reader.py` and adding `open_premises_analysis/`.

Version 2.0 (26 November 2026) is the public release replacing version 1.9.2 of 16 November 2026; it folds together the private revisions 1.9.3, 1.9.4 and 1.9.5 (summarized below) with four verified additions, and neither headline value of $1/\alpha$ moves (137.0246 ± 0.0072 and 137.0193 ± 0.0073, −1.58 and −2.30 errors). First, in section 2.5, the count of a two-cycle closed history of one field's quanta is sharpened: counted once without splitting off the longest stretch as the line, the textbook gauge weight leaves a mass in every lawful gauge, and taking the meeting rule whole removes the mass only while running at $-\frac{76}{3} = \frac{19}{17}(-\frac{68}{3})$, confirming that the longest-stretch split is needed for the running as well as for no mass. Second, section 2.5 adds the native shared-link computation of the pure-link two-loop coefficient $A = \frac{34}{3}$ ($\frac{19}{2} + \frac{11}{6}$; mechanism split $6 + 6 - \frac{2}{3}$; own-turn rate polynomial $A(r_t) = \frac{49}{12}r_t^2 - \frac{1}{2}r_t - 4$; corner logarithm proportional to $r_t(r_t - 2)$ with the four-link meeting term), closing the two-loop slope ratios $\frac{102}{121}$ and $\frac{26}{49}$ natively while stating all five limits plainly. Third, section 3 adds the complete four-loop three-coupling check at first-order cell matching on the 2024 inputs and the reconciliation of pure $\overline{\text{MS}}$ and hybrid low-end electroweak determinations (97 per cent of their split traced to the Fermi-constant relation; 2025 `SMDR` v1.3 fit landing colour, weak and hypercharge at −0.40, −0.81, −0.35; complete four-loop shifts at most 0.044 errors; sharp hadronic window

straining every dispersive e^+e^- evaluation in the 2024 review outside two errors on the hypercharge row). Fourth, section 5 removes a redundant restatement of the carrier-imbalance weak-share landing as a cost-coefficient measurement and adds the three-loop own-answer combinatorial check (1/97 against 2^{-7}, with odd balanced-cell denominator $4k + 1$). The code archive is updated to version 2.0, patching `colour_turns_24_bonds/README.md` to match version 1.9.5's colour-ground correction and adding `pure_link_two_loop_A/` and `four_loop_three_coupling/`.

Version 1.9.5 (16 November 2026) replaces version 1.9.4 of the same day. It corrects an error; no number of the result moves, and the code archive is still version 1.8. Said first: since version 1.8 the paper said that colour's carry from the cell rests on the same premise as light's read, that the links' own turns are not summed as weights at the cell, and since version 1.9 that a statement of that premise on the record would settle both. That is wrong: that premise does not settle colour's carry. With the turns not summed as weights, the pair alone still does not keep colour's links near no turn: on the 24 bonds the mean of $\frac{1}{3}\operatorname{Re}\operatorname{tr} U$ over triangles in the least state is -0.27, -0.30 and -0.32 at K, twice and four times that, on the half-turn side. That the pair alone leaves a triangle's mean reading free, anywhere from $-\frac{1}{3}$ to one, is proved on one triangle; which side the least state takes is computed, on one triangle and on the 24 bonds. Colour's carry from the cell therefore rests on a second open premise, which the record does not state: that colour's links sit near no turn at the cell. Colour's landing at $4K$, the strong coupling solved from colour at the cell, and the W mass and the mixing angle predicted with that coupling rest on that premise. K, light's count and its value, and the two values 137.0246 ± 0.0072 and 137.0193 ± 0.0073 do not depend on it. Wordings: section 2.4 says this in place of what it said, and states the premise; the chain table replaces the clause that put colour's carry on the reading, and gives the premise its own row, which section 8.1 lists; section 5 says that colour's landing, the strong coupling solved from colour at the cell, and the W mass and the mixing angle rest on the premise; section 4 says the same of the strong coupling taken from the ledger; the abstract and section 4.1 say that colour's place in the coarse structure rests on it; the odds stand as computed; the abstract, where colour lands, and section 2.2, where colour confirms the count, name it; and the notes on versions 1.9 and 1.8, which said that colour's carry rests on the one premise, keep what they said and point to this correction.

Version 1.9.4 (16 November 2026) replaces version 1.9.3 of the same day. No number moves, and the code archive is still version 1.8. Wordings: the abstract gives the three couplings' chi-square as 4.70 and 11.94, without the starting value, 1.40, that the body never named, and section 3 gives it after the table of the asks; section 2.4 names both discrete alternatives the chain table counts for the cost's coefficient, where it named one; section 2.5 says that the distances of seven and nine postings are counted on the logarithm of the end, where the error is the same on both sides, and that the ± 0.16 is that error carried to the postings at their centre; section 3 says that every part of the matching's second order is computed and names the line for what the count leaves open, where it said "as far as it is computed", and says that the revision of the reference state, taken as version 1.0 took it, is a line of 0.000573 that equals light's first-order error to the digits printed; section 5 corrects a comma; section 6 says that its range of 2.9 to 5.3 and 3.3 to 6.0 errors covers the six electron–positron determinations of Table 4 with errors of 0.0001 or less; the chain table and section 8.1 add a fifth tag, *located, not adopted*, for the one row whose form was found with the measured share in view and is not

the adopted one; section 8.1 says that its list holds every row of the chain table whose step is not derived, each once, where it said that every such step is a premise, says that every other listed row is a premise, names the two rows whose notes qualify their tags, and says that the last row's first half is the pair reading listed above it; the appendix says that its brackets give the file, with the label of its line where the file labels its lines, and adds `scalar_count`, C5, to the bracket for the one sixth that enters in part 3; and the reference to Eddington's 1929 paper gives its DOI.

Version 1.9.3 (16 November 2026) replaces version 1.9.2 of the same day. No result moves. Said first: a review of version 1.9.1 found a sign inconsistency in the cost of a complex ratio, and it is real in the paper's wording. Section 5 printed that cost as $J(z) = (z + 1/z)/2 - 1$; on a pure turn $e^{i\varphi}$ this is $\cos\varphi - 1$, which is never positive, while section 2.2 costs a loop's turn at $1 - \cos 2\theta$. Two things had one name, and they are now named apart: the cost and the read. The cost of a comparison is never negative: J on a size, $1 - \cos$ on a turn, and $|z - 1|^2/(2|z|)$ on a comparison z that carries both; the count of four rests on it. The read is the mean of the two readings, $(z + 1/z)/2$, and it is what light reads: a size raises it above one by its cost, a turn lowers it below one by its cost, and on a bond that carries both its real part is the product $\cosh u \, \cos\varphi$, which is where the dressing comes from. The review's remedy, one nonnegative cost in place of both, is not taken: that cost has no product of the size and the turn, so it would leave light undressed, at 8 in place of $8K$, 564 and 563 errors low. With the two named apart, no number moves.

What a reader needs to judge the result is now said plainly. Section 8.1 lists every premise, tagged proved, checked, working or borrowed, and the abstract and section 1 say once that the result is computed under the readings listed in section 8. The chances of section 4.1 are named for what they are, bookkeeping of the combinations tried and not tests of significance. The weak share's form is called a located form that lands, not an independent prediction. Section 1 names the result's dependence on the hadronic input: within two errors on most lattice determinations, strained or failing on the electron–positron evaluations with errors of 0.0001 or less. Section 3 and the carry table now separate light's error within one reading set from the spread across readings. An appendix now rebuilds the two values part by part: for each part of the matching's second order it gives the weight and its source, the count's sums, the two constants and the shift of light's ask; every decimal number in it is printed by the code archive, and it closes with the sum.

Corrections of version 1.9.2, none a number of the result: section 2.1 said each event sits on thirty-two triangles, where ninety-six triangles have a corner at each event and there are thirty-two triangles for every event; two sentences of section 2.5 quoted the coefficient's shortfall as 2.2 where the paper prints 2.1, and gave the one combination of the couplings that does not involve the end as 3.5 where the table of section 8 has 3.4; section 4.1 called the last chance the package's own with nothing left to choose, where the weight's coefficient, one or two, is still chosen; section 2.3 said the identity on the weight holds for each bond, where it is exact for the average over the bonds and holds for each bond only in the limit; section 2.5 wrote the cost of a comparison z read on its pair as $(z - 1/z)^2/2$, which is negative on a turn, where it is $|z - 1/z|^2/2$; the table of section 2.5 printed its first slope as -969.00, five figures where its source prints four; and section 3's display of the slope divided 10.888 by 137.036, which gives 0.0794536, where the slope it prints, 0.0794553, is 10.88823 over 137.036.

Wordings: In section 5, the sentence that said $K/16$ uses no stay now speaks of the carrier's imbalance alone; the chain table's row for the carrier's form says it is the split number p that comes out low; each of the three forms is said to need at least one statement the record does not now make; the adopted form is said to be the first written; and the mean of a ratio's two readings is called that, not its cost. Section 5 names the stay's two derived steps and its two readings; the W mass and the angle are said to barely tell the package from the Standard Model; the sentence on free fall names the two forms that read constants of the carrier, where it said all three did and that free fall cannot reach them, and says that clocks bound the dressing's response to the potential; "reads no dressing" is now "hardly depends on the dressing"; the stay's ratio is written r, since c is light's speed; and the abstract and the W and Z sentences, which called the open choice carried in the error a tie, now say what it is: which of two equal stretches is the line. The carry table's first line now lists its inputs, with the sources of the top's, the scalar's and the other quarks' masses and of the hadronic input, and section 3 says that solving the carry in full instead of through its slope moves the two values by about one and three millionths, and no value of α^{-1} in the results, the table of the carrier's reads or the table of determinations changes in a printed digit. Section 2.5 now gives the displayed inequality for $-\ln(1-q)$ its hypotheses in place of "small t"; and says that the pole of the top's weight with the scalar has no product term by itself and becomes the published pole with the stretches' own lines. It says that the four statements not yet derived from the ledger's foundation carry the matching at first order, and names four further readings that the table of section 8 marks: one on which the first order also rests, and three at second order. It also calls half the difference of a part's two constants the shift of the end, apart from the shift of light's ask, and writes that half difference δ, since c is light's speed. The chain table's row for the pair reading now writes $J(z^2)$ as "the cost of z^2", the phrase section 2.2 uses, and section 2.2 gives the one-sided reading as the cost of the pair's square root, z, where it wrote $J(\sqrt{d/c})$ without saying what d and c stand for. Section 2.1 gives the twenty-four-neighbour bound its condition; section 2.3 says how the carrier's uniform shift is fixed, which leaves $N-1$ directions; the reference to Eddington's *Fundamental Theory* gives its OCLC number, the reference to Enz's biography gives its DOI, and the references to two chapters of the Review of Particle Physics give each chapter's number after its title; and the abstract says the muon's strain holds under both statements. The code archive is now version 1.8: it prints every figure of the appendix and the split of light's error, names the exact package versions its outputs were made with, and reruns every folder with one script.

Version 1.9.2 (16 November 2026) replaces version 1.9.1 of the same day. Said first: the top quark's closed history with the scalar inside, which versions 1.7 to 1.9.1 carried in the error at half an e-fold, is computed, and light's part of it ends 0.237 e-folds below the scale. The two values move up in their last printed digit, to 137.0246 ± 0.0072 and 137.0193 ± 0.0073, and stay 1.6 and 2.3 errors below the measured value; the three couplings' chi-square falls from 4.79 and 12.06 to 4.70 and 11.94. With it every part of the matching's second order is computed, and the one line left in the error is for which of two equal stretches of the links' own quanta is the line. The weight for the part is derived from the action and checked against the published two-loop running for every pair of charges; a second, independent derivation gives the same weight and sums, and with the same reading of the scalar's own line the same constants; the one other count possible in form gives the quark a mass and is not taken. Corrections of version 1.9.1: two figures were printed rounded twice, each one unit off in its last digit

and neither a number of the result: the error of the hadronic input asked with the ledger's inputs under the first statement, printed 0.000010 where one rounding gives 0.000009, and the pull of the determination from e^+e^- data below 2 GeV, printed -5.31 where one rounding gives -5.32; both now stand on the new runs. The closing section called light's shortfall, 1.58 and 2.30 of its errors, *inside the carry's error*; light's errors are the carry's, so the clause was false, and it is cut. The chain's table gave the stay's step as derived, with $c = 12/16$; as the body says, it is a reading, adopted with the measured share in view, and $r = 3/(K_m + 3)$, three quarters only when nothing is dressed. Every number that rests on the runs is restated on them, most in their last digit. The stay is now defined as the share of a record's moves that rest, as in the cited law of motion. The code archive's version 1.7 adds the computation of this part and restates every folder whose printed numbers move.

Version 1.9.1 (16 November 2026) replaces version 1.9 of the same day. No number of version 1.9 was wrong, and no result moves. The code archive's version 1.6 computes the second statement's count, which version 1.5 took as an input. The labels, wordings and rows follow. One quantity was printed two ways: the gap between the hadronic input the weak share asks by itself and the one light asks stood in section 5 as a bound, within 1.6 and 2.8 combined errors, rounded up from 1.56 and 2.71, and a paragraph later as the value, 1.6 and 2.7; it now reads as the value in both places. Every chi-square the paper prints counts the three couplings' correlations, and the paper now says so; the values stand (4.79 and 12.06 are computed from the unrounded pulls and correlations). The carry's errors already include the dressing's own, and the sentence that read as adding it again now says so; the pulls stand. The sentence on the first-order matching no longer says the comparison is made at that order: the result is compared with the second order included. One more charged fermion at 1 TeV shifts each value by 7.764, now printed to that digit so that the values with it, 144.79 and 144.78, add up, and the errors it moves light by are given as about 1,070 in both places. The one combination of the couplings that does not involve the end is said to measure the same shortfall as the dressing and the coefficient: three fits of one shortfall. The chain's table has three new rows, each a reading: how an exchanged link meets a history, the count where the links' own quanta meet one another, and where a moved term is counted, which gives the two values. Where the paper says the matching's second order is derived, it now names the readings it is derived under. The table's status *checked* is split: *selected* where the couplings themselves miss the alternatives, and *checked by measurement* where another measurement does. The quarter that misses the weak share by 4.7 and 4.2 errors is named as what it is, a quarter of the rest's weight. The reproduction command copies as printed, Enz's name is given in full, every web address extracts on one line, and $1/e^2 = 1/(4\pi\alpha)$ is defined where it is first used.

Version 1.9 (16 November 2026) replaces version 1.8.1 of the same day. Said first: the result is now quoted as the two values the record leaves, side by side, each with its own error, 137.0244 ± 0.0072 and 137.0192 ± 0.0073, below the measured value by 1.6 and 2.3 errors. One statement the record does not make decides between them, and section 2.5 gives both statements exactly. Version 1.8 quoted 137.0199 ± 0.0094, taking the mean of the two sides of each jump, a convention, as its centre; that centre is retired, and every number that depends on the choice is given for both, the first statement's first.

Corrections of version 1.8.1. Besides the three bounds it corrected itself, version 1.8.1 printed four more rounded to nearest rather than the safe way, each off by less than one unit of its last digit and none a number of the result: one end for every spin "within 1.8 of the error" (exact

1.8004), one charged fermion "anywhere below 0.957 of the cell's energy" (exact 0.95653), the room's lower end −0.00003 (exact −0.0000262) and the share's own ask "within 1.5 combined errors" (exact 1.53); this version prints every bound the safe way. The third filing, made with version 1.8, likewise prints its bounds at an error of 0.00010 to nearest, 0.0276054 and 0.0282722, a few billionths outside the exact lines 0.02760538 and 0.02827222; a filing stays as filed. In section 3 the first-order errors of light and of the weak share were version 1.6's (0.000572 and 0.0000583; now 0.000573 and 0.0000584, as the carry's lines give them). In section 5 the W-mass combination with the CDF measurement was cited to that measurement alone, and its distance, 3.4, was the filed W's, printed after the second-order sentence; the combination is now cited to its own paper, at its own digits, with the second-order W under both statements. And version 1.8.1's note named only the carrier's own stiffness among the earlier values its corrected odds had used; the mean loop weight was corrected with it, from 1.040851 to the printed 1.0408495.

Also new. The abstract and section 1 say that no continuous number is fitted to light and, beside it, that the readings were named with the measured couplings in view. Section 1 says that the result sits with the lattice determinations of the hadronic input, and that the carry uses the Standard Model's fields alone between the Z and the cell; section 3 gives what one more charged fermion would do. Section 2.4 names the one open premise that the read and colour's carry both rest on (version 1.9.5 corrects that, and colour's carry rests on a second open premise, that colour's links sit near no turn at the cell), and section 7 opens with what falsifies the closure. Figure 1 draws light's two asks under both statements: on the lattice hadronic input 1.04051 ± 0.00007 and 1.04056 ± 0.00007, on the data-driven input 1.04081 ± 0.00014 and 1.04086 ± 0.00014. Table 1 gives the carry from the measured inputs to the asks at the cell, step by step, under both statements, and Table 3 sets side by side the three forms that land the weak share. The references give their identifiers, and the two chapters of the Review of Particle Physics are cited with their authors. A paragraph says how to reproduce the numbers from the archive.

Version 1.8.1 (16 November 2026) corrected version 1.8 of 15 November; no number of the result moves. Figure 1 drew light's two asks at version 1.1's values (on the lattice hadronic input 1.04048 ± 0.00007, on the data-driven input 1.04078 ± 0.00014) while its caption gave version 1.8's; it now draws version 1.8's, 1.04056 ± 0.00009 and 1.04086 ± 0.00015. Both figures are now drawn directly from the numbers the paper prints, with no coordinate set by hand.

The other corrected numbers now read as version 1.8's run gives them; each is given here with the value version 1.8 printed. Light's number of senses in the chain's table (section 8) is 4.0006 (printed 4.0005), as section 2.2 already said, and colour's is 4.003 there and in section 9 (printed 4.002). Three bounds were rounded the wrong way: the lattice determinations lie within 1.8 errors on four of the five (printed 1.7; one lies at 1.71), in the abstract and in section 6; figure 2's filled circles lie within 2.6 combined errors (printed 2.5; one lies at 2.51); and the muon's passing range at this version's error is 0.02757 to 0.02790 (printed 0.02756 to 0.02791, rounded outward). Restatements of the asks that had kept a first-order or version 1.7 value now stand on this version's run. If the cell's bit counted the record's own stiffness only, light would stand at −2.20 (printed −1.77; sections 2 and 8). Light less twice colour, which contains no dressing, is measured at -0.0042 ± 0.0099 (printed -0.0034 ± 0.0098; sections 2.2 and 2.5); through it the coefficient is located at 0.964 ± 0.085 (printed 0.971 ± 0.084) and the end at 0.98 (printed 0.99) in units of $\hbar c/\lambda$. With the weak share in, the neighbouring end gives $\chi^2 = 16.1$ for three (printed

19.9) and still does not land. Within two errors the closure leaves room for a relative addition between −0.00003 and +0.00033 (printed −0.00007 and +0.00022; sections 2.5 and 8). Colour's window for K is ±0.0013 (printed ±0.0012; section 5). The harmonic mean misses light by 17 errors (printed 15; sections 2.4 and 8). For heavy neutral fields the sum of $y^2 \ln(E/M)$ stays below about 9 (printed 11; section 3). In section 2.4 the computed alternatives still carried version 1.7's misses of light: one link's weight misses by 5.7 to 7.4 errors (printed 4.7 to 6.2), the geometric mean by 9.6 (printed 8.1), the harmonic mean by 17 (printed 15) and the product by about nine hundred (printed eight hundred). In section 4.1 some figures were computed with the earlier value of the carrier's own stiffness, 1.040747, where the paper prints 1.040759, and now use the printed one. With the correlated chi-square as the measure the second of the three chances is 0.038 (printed 0.039). The ten-choice menu gives 0.0065 (printed 0.0066), in the table and in the text; the menu with the hadronic input also a choice gives 0.042 (printed 0.043) and the wider menu 0.076 (printed 0.077). On the errors without the line the second chance is 0.051 (printed 0.052). On the ten-choice menu the couplings put 0.94 of the weight on pure logarithms, 0.62 on the own stiffness and 0.28 on the long-wave value (printed 0.95, 0.68 and 0.24).

In section 7 the muon paragraph's first computation is now named for what it is: it uses the value filed with version 1.1, not this version's ask, which the paragraph gives next. The note on version 1.7 below again prints version 1.7's own count, 4,883 errors of two events, where version 1.8 had put its 5,547. Section 2.5 now prints the second order's constants in one place: for each of its five computed parts the constants under the count and under minimal subtraction, where its logarithm ends, light's slope and the shift of light's ask, with the sum that rebuilds the result from the printed numbers. The code paragraph named archive version 1.2; this version comes with archive version 1.4, and the paragraph now says what versions 1.3 and 1.4 added.

Version 1.8 (15 November 2026) replaced version 1.7 of the same day. Four of version 1.7's figures were wrong, and are corrected here first. Section 2.5 said that the two parts of the second order not computed, added plainly instead of in quadrature, would give 0.0120 in $1/\alpha$ and 0.0140 in all; those were the plain sum of all four parts then not computed, and for the two it named the plain sum is 0.000889 in light's ask, 0.0112 in $1/\alpha$ and 0.0133 in all (the quadrature figures, 0.0079 and 0.0107, stand). The chance of an accidental fit with the search, on the menus fixed before the count, was 0.021, not 0.022, wherever it was printed (the abstract, section 4.1 and the note on version 1.7). The sentence on the cycles of three postings, and the abstract's one form that fits slightly better, carried version 1.6's figures: on version 1.7's asks the cycles were at −0.84, −0.34 and +0.01, and four of the thirty-two forms fitted better, all of them walks on the bonds, so the exclusion and its conclusion stand. The Z line was computed with the dressing filed with the prediction, 1.040399, rather than the paper's 1.040396; on the paper's it reads 91.1968 GeV, 9.2 MeV above the measured value and 0.87 combined errors (printed 91.1966, 9.0 and 0.85). The matching's second order is now computed where the record settles it: the histories in which the link that is read meets the exchanged link's own line, the scalar rings, and those in which the links' own quanta meet one another, whose count follows from two facts the record fixes and leaves two choices open, carried in the error. With them the result is $1/\alpha = 137.0199 \pm 0.0094$, and the measured value lies 1.7 errors above it (version 1.7: 137.0234 ± 0.0107); every number that rests on the asks is restated on them. One pole term of a published two-loop weight is recomputed (section 2.5).

Thirteen more values were set against the weak share since version 1.7 (eighty-eight in all, fifteen within one error on this version's ask). The far-share form's condition is stated: it holds while a transfer weighs one. Colour's carry from the cell is said to rest on the reading that the links' own turns are not summed as weights there; version 1.9.5 corrects that, and colour's carry rests on a second open premise, that colour's links sit near no turn at the cell. On this version's ask the muon's leading hadronic part strains. The value of the hadronic input filed in advance stays and is scored as filed; a new value is filed with this version, and both are kept. The weights of the ten-choice menu now lean to the carrier's own stiffness over the long-wave read that the derivation takes.

Version 1.7 (15 November 2026) replaced version 1.6 of the same day. More steps of the chain are marked checked, each with its alternatives counted and missed by measurement: the matching's unit (one value, 71/144, is counted and not missed, and one half is the ledger's own value), the summed count of an unrecorded history, gravity's strain, the cost as a weight at coefficient one, and a record holding each link's unseen part; the cell's one bit a face is checked through the ledger's law for the electron's mass, which is fixed on one datum. The 4,883 errors of two events are the sum applied inside one entry; the record's own sums add postings, and that is the unit. The weak split's number is stated as three located forms, level, each needing one statement the record does not now make (for the carrier's form, one weight on every triangle), and free fall shows that it is not a local stay. A correction: the count of values within one error of the share, printed as eight since version 1.2, was eleven on the error printed then; on this version's error it is twelve of seventy-five, and three of the twelve have a reading behind them. Light's shortfall is named as a tension. The matching's second order is given its form: an exchanged link meets a closed history as the link that is read does, a working reading; the link's own stretch is counted as a line of its own quantum, the only complete count of the three; and where a field's quanta meet one another, the record fixes a condition the count must meet, and which count meets it is not yet settled. The 2026 lattice determination's perturbative piece was recomputed with the code it names (section 6). The matching's second order is derived in form where the record settles it and computed in part; with it the result moves to 137.0234 ± 0.0107, and the parts not computed are named and carried in the error. The weight of the agreement is recounted on the new errors: 0.21 given the coarse structure, 0.022 before the count, 0.00002 after the derivations.

Version 1.6 (15 November 2026) replaced version 1.5 of the same day. Version 1.5 called the counts readings identified with the couplings in view and left open whether an entry's two sides are priced once. That question was framed wrongly: an entry is one thing, one comparison of its two sides, and is priced once; asking whether its debit and its credit are one event or two adds a split the entry does not have. The counts follow from it, and the chain's table marks them checked. Every other pricing misses, two events by thousands of errors and nothing by all of light's stiffness; the measured couplings over K, 8.0006 and 4.0019, confirm it. It was first found with them in view. The matching now says that the whole-cycle form is the posting law's own, that the earlier attempt's one half is a field's cost and not the unit's, and that the dressing left free and the coefficient light locates show one shortfall. No number of the result moves.

Version 1.5 (15 November 2026) replaced version 1.4 of the same day. Version 1.4 said that, of the values the record already held for a posting's cost, the nearest other was a cost of $\ln \phi$ and every other missed by twelve errors or more. A fuller search of the record found one value

the couplings cannot tell from the half, 71/144, and two at about eleven errors. No number of the result moves.

Version 1.4 (15 November 2026) replaced version 1.3 of the same day. Version 1.3 said that the record had left the strain a posting presents open between two values, that at the golden strain it priced a debit with its credit in one delta, and that it held nine numbers for a posting's cost. Each was wrong about the record. The record names more values, each fixed by one definition and none derived. Among them one posting costs one half where a debit posting with a credit posting costs one, which is the unit adopted here. Its price of one delta is for postings on one side of one account, not for a debit with its credit. The open step is stated sharper: if the cost of a record obeys the record's law of composition, an entry's two sides held in one record cost nothing or $J(z^2)$, never the sum, and the count of four rests on pricing one record, a reading the record itself leaves open. No number of the result moves.

Version 1.3 (14 November 2026) replaced version 1.2 of the same day. Version 1.2 said that the number of the matching's unit was newer than the comparison at the cell and that the one older number for a posting was $\cosh 1 - 1$; that was wrong, since the record held the unit's number before the end was located. Light's four and the unit's one half were stated to rest on one reading, the comparison of an entry's two sides, whose credit side is the whole rest of the record because nothing less balances the entry. The complex cost was written with its constant. No number of the result moved.

Version 1.2 (14 November 2026) replaced version 1.1 of the same day. The carry's three-loop completion is now taken from a shared start (+0.000005 in light's ask, not +0.000019), which moves the result to 137.0279 ± 0.0072 and light to -1.13; the filed input stands. The weight of the agreement now counts every form tried for where the logarithms end (0.014 before the count, 0.15 given the coarse structure), says that one of them, cycles of three postings, fits slightly better, and says why double entry excludes it. The pair reading is stated in the record's terms and closes colour beyond light; the matching's unit is set by the posting law, not by the pair; the division of light between the weak link and hypercharge is given its meaning, the readers' mean split number; the term of the cost that tells hypercharge's turn from the weak link's is identified, and a form built from the carrier alone lands the weak share at -0.49, one of three forms that land, with the count of what was tried; the matching's second order, the cell's one bit a face and the inside of a history that leaves no record are named; the end the three couplings ask together is given.

Version 1.1 (14 November 2026) replaced version 1.0 of the same day. The carry's error budget was recomputed with signs: one line of version 1.0's budget, the revision of the reference state, had been added in quadrature over two couplings whose moves cancel in light, so light's error is 0.000572, not 0.000811, and three computed shifts are applied. The dressing was computed a second time, independently, and the headline uses that computation. Together these moved the result from 137.0281 ± 0.0102 to 137.0277 ± 0.0072; on this budget the reported package lands light, colour and the weak share within two errors, not one. The census of the hadronic input was completed: two lattice-based determinations lie above the value the closure asks, 0.02794(9) and 0.02795(11), and the sharpest electron–positron evaluations lie more than three errors below it. Colour's count is read on the pair as light's is, and the factor two between them is the trace of the generator. The second-order value 1.040378 is the one with a Gaussian covariance of the weights. New in version 1.1: which stiffness light reads, from

what a coupling is, and the readings under the weighting and the matching; the weak share's count and the open problem it leaves; the count of the trials, with the chance conditioned on the coarse structure; two numbers filed before comparison and the solved Z mass; the muon equation; a second filing of the hadronic input; and the table of the chain's steps, each marked derived, reading or borrowed. The filing of version 1.0 stands with its numbers.

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