

The Fine-Structure Constant from the Recognition Ledger

Light's stiffness at the smallest cell, carried to rest

Jonathan Washburn

Recognition Physics Research Institute, Austin, Texas

jon@recognitionphysics.org

Abstract

With the matching between the ledger's lattice and the continuum derived at second order and computed, the fine-structure constant the recognition ledger gives is $1/\alpha = 137.0199 \pm 0.0094$, 1.7 errors below the measured value. The error carries the choices the record does not make in the count of the links' own quanta, at a jump of its weight and at a tie; across their readings the result runs from 137.0146 to 137.0260, the measured value above it in every reading, by 1.4 to 3.0 errors, and the centre, the mean at the jump, is a convention. Colour lands at −0.54 errors and the weak share at −0.50, and the three couplings' chi-square rises from 1.40 to 3.43. If the 2026 lattice determination's perturbative piece is as our recomputation finds, the result is 137.0294, 0.69 errors from the measured value; the paper keeps the determination's printed value.

The fine-structure constant $\alpha \approx 1/137$ sets how strongly light and charge act on each other. It is measured to one part in ten billion; no accepted theory computes it. Here it is computed from the recognition ledger, with no continuous number fitted to light.

At the ledger's cell, $2\sqrt{\ln 2}$ Planck lengths on an edge, light's turn stiffness $1/e^2$ is a whole count, 8, times the stiffness a long wave reads from the ledger's own carrier, $K = 1.040396 \pm 0.000002$. It is computed twice, independently, on the lattice of events with no measurement in it. Carried to rest by the Standard Model's computed running, $8K$ gives

$$\alpha^{-1} = 137.0199 \pm 0.0094,$$

against the measured 137.035999177(21): a hundred and eighteen parts per million below, 1.71 of the error. The matching's second order is derived where the record settles it and computed from published two-loop weights with no further choice; what the record does not fix, the top quark with the scalar, which is small, and how the count of the links' own quanta treats its jumps and ties, is in the error. Version 1.7, which computed one part of it, gave 137.0234 ± 0.0107, 1.17 errors below the measured value. Colour lands with the same dressing (−0.54 standard errors) but admits every read of the carrier; the weak share (−0.50) tests its own form, not the dressing, and two other forms land it too, one built from the carrier alone (−0.67); the quarks' factor of three (−0.63) rests on fitted terms.

Light's read follows from what a coupling is, and the matching has no free constant if unrecorded histories are summed over every whole count: none at first order, and at second order the one the count gives. The counts follow from what an entry is, one comparison of its two sides

priced once: other pricings miss, two events by thousands of errors, and the measured couplings over K, 8.0012 and 4.0026, confirm it. It was first found with them in view. The other premises that carry the physics are readings identified with the couplings in view. The weight of the agreement is the coarse structure: light and colour at whole counts times one stiffness near 1.04 at a cell fixed by gravity, which a broad null gives once in 20 to 1,200. Counting every form tried for where the logarithms end, the precision adds little: some candidate fits all three couplings as well with chance 0.31; on the menus fixed before the count the chance is 0.041, falling to 0.00005 as the derivations found afterward are granted. Four forms tried fit better, all of them walks on the bonds, which the record excludes.

On the hadronic input $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$, which carries most of the error, the closure lands within 1.8 errors on four of the five lattice-based determinations (one on its ask) and 2.5 below the fifth; it is strained or fails on every electron–positron evaluation with an error of 0.0001 or less, by 3.0 to 4.6 errors. The value it asks is filed as $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027882 \pm 0.000019$ (version 1.0 filed 0.027878 ± 0.000056); no determination made public since has scored it. With the second order computed it asks 0.027939 ± 0.000048, filed with this version; on it the muon's leading hadronic part sits 2.3 errors high, a strain (2.1 to 3.2 over the readings); on the perturbative piece as we recompute it, 1.5 to 1.6, inside the rule. On the earlier filing it sat 2.0.

Version 1.8.1, 16 November 2026. What changed is noted at the end.

1 The number

In 1916 Arnold Sommerfeld worked out how relativity splits the spectral lines of hydrogen into close pairs, the fine structure [1]. The size of the split was set by one pure number, which he called the fine-structure constant, $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$. It has no units. A civilization that counted in other units would find the same value.

Dirac's equation of 1928 explained the fine structure exactly [2], and it explained it in terms of α; it did not explain α. Arthur Eddington spent the rest of his life trying to: in 1929 he argued that $1/\alpha$ is exactly 136, and later that it is exactly 137 [3, 4]. Wolfgang Pauli worried at the number for decades; he died in 1958 in room 137 of the Red Cross hospital in Zurich [5]. In 1947 Lamb and Retherford measured a shift in hydrogen's levels [6], and Kusch and Foley an excess in the electron's magnetic moment [7], that Dirac's theory did not contain. Schwinger, Feynman and Tomonaga computed both from quantum electrodynamics, starting with Schwinger's $\alpha/2\pi$ [8]. Every one of those computations takes α as given. Feynman, who did as much as anyone to make them, wrote in 1985: "It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man" [9].

Measurement went on without explanation. The quantum Hall effect gave α from a resistance [10]. A single electron held in a trap gives it from the electron's magnetic moment, now to 1.1×10^{-10} [11]. Atoms recoiling from light give it from the ratio of Planck's constant to an atom's mass [12, 13]. The value recommended in 2022 is

$$\alpha^{-1} = 137.035999177(21) \quad [14].$$

At the energies of the large colliders the coupling was seen to depend on the length at which it is read: about $1/128$ at the mass of the Z boson [15]. That dependence is computed. The value at rest is not. In the Standard Model it is an input.

This paper computes it. The result is $1/\alpha = 137.0199 \pm 0.0094$, and the measured value lies 1.7 of its errors above it. The matching between the lattice and the continuum is derived at second

order where the record settles it, and computed there from published two-loop weights with no further choice: the histories in which the link that is read meets the exchanged link's own line, the scalar rings, and those in which the links' own quanta meet one another. The count of the last leaves two choices the record does not make: how end terms count at a jump of its weight, and which of two equal stretches is the line. They are carried in the error; across their readings the result runs from 137.0146 to 137.0260, and the measured value lies above it in every reading, by 1.4 to 3.0 errors. The centre takes the mean at the jump, a convention. The top quark with the scalar, which is small, is not computed, and it is in the error too. Version 1.7, which computed one part, gave 137.0234 ± 0.0107.

Section 2 states what the recognition ledger gives for light at its smallest cell, which of its steps follow from stated premises, and which are readings. Section 3 carries the measured couplings from rest to that cell. Section 4 compares the two, states the fine-structure constant the ledger gives, and counts what the agreement weighs. Section 5 does the same for colour, for the weak share and for the quark masses, and gives two numbers the package was never compared with before they were filed. Section 6 says where it does not close. Section 7 files the numbers that test it. Section 8 lists every step of the chain with its status.

2 Light at the ledger's cell

2.1 The ledger, its bonds and its cell

The recognition ledger records every event as a posting on one of four accounts, each with a debit and a credit column [18]. Counted on both columns, the events form a lattice in four dimensions, the integer points together with the integer points shifted by $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. Its twenty-four shortest vectors, the bonds, are the sixteen half-integer vectors $(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$, which are the eight debits and the eight credits, and the eight unit vectors $\pm e_i$, which are a debit with its credit on one account, its reverse, and the six edges of the record cube [18]. All twenty-four have one length, and the lattice's symmetries carry any bond to any other. Every event has twenty-four bonds and every bond joins two events, so there are twelve bonds for every event. Every bond lies in eight triangles of bonds, each event sits on thirty-two triangles, and every triangle closes one unit bond with two half bonds, one a debit and the other a credit, so that each loop is one recognition closed by a unit step; neither kind of bond makes a triangle by itself. No arrangement of points in four dimensions has more than twenty-four nearest neighbours [21], so twelve bonds per event is a count of this lattice and not a setting; the smallest change a lattice allows, one bond more or fewer per event, would move light at the cell by twenty-nine of its errors or more.

Light is the loop law on the triangles of those bonds: a link carries a turn, and light's coupling is how stiff that turn is. Colour is the same loop law on an account's three partners.

The lattice has a size. One bit of record per λ^2 of area gives Newton's constant as $G\hbar = \lambda^2 c^3/(4 \ln 2)$, so the cell's edge is

$$\lambda = 2\sqrt{\ln 2}\, \ell_P = 1.665109\, \ell_P \quad [19],$$

and the energy at which light is read at the cell is $E_P/1.665109 = 7.3322 \times 10^{18}$ GeV, with $E_P = 1.22089 \times 10^{19}$ GeV the Planck energy. The edge is fixed on gravity's bit and is used here

unchanged: it is taken from the measured Newton's constant. Under the count of section 2.5 the closed histories of the known fields themselves supply 1.57 per cent of gravity's stiffness, and the cell's one bit a face is read as counting the whole stiffness, these included; were it to count the record's own stiffness only, the cell would be 0.78 per cent larger and light's pull −2.20. Where, at that edge, a coupling is read is the matching's question (section 2.5).

2.2 The counts

At the cell a link's turn stiffness, before anything dresses it, is a whole count, and the count is set by the lattice once two things are said: how a loop is read, and on which body. On the twenty-four bonds, for every uniform field strength the squared loop values of the triangles (the field strength's flux through each) sum, per unit four-volume, to exactly $F_{\mu\nu}F^{\mu\nu}$; the triangles' second moment is the same in every direction. So if a loop costs $S \cdot \frac{1}{2}\theta^2$ for a small turn θ in each direction in which the hopping body turns, each loop is dressed by K, and the body's generator squares to trace T, then the lattice's stiffness equals the continuum coupling for every field exactly when

$$\frac{1}{g^2}\Big|_{\text{cell}} = 2\,STK.$$

Colour is read on an account's three partners, where each generator has $T = \frac{1}{2}$. Light turns one of a body's two columns and leaves the other still, so its generator is a projector of rank one and $T = 1$. The weak link and hypercharge, read on the same two columns, have $T = \frac{1}{2}$ each, and light's trace is their sum, $\operatorname{tr} Q^2 = \operatorname{tr} T_3^2 + \operatorname{tr} Y^2 = \frac{1}{2} + \frac{1}{2}$; that is the linear algebra of light's trace, not a statement that each part has $4K$ (section 5). The ratio of light's count to colour's is therefore the ratio of their generator traces, two, for every S: a unit charge against the fundamental's one half, as it is between the $U(1)$ and $SU(N)$ loop actions of field theory at equal cost per turn.

What sets S is how a comparison is costed. The reading adopted here costs it on its pair, the debit reading over the credit reading. For a comparison z whose credit reading is z^{-1} the cost is $J(z/z^{-1}) = J(z^2)$, one formula for every comparison. For a loop the credit reading is the loop's reverse, so the pair reads the turn twice,

$$1 - \cos 2\theta = 4\,(1 - \cos\theta) - 2\,(1 - \cos\theta)^2,$$

and $S = 4$:

$$\frac{1}{g_3^2}\Big|_{\text{cell}} = 4K, \qquad \frac{1}{e^2}\Big|_{\text{cell}} = 8K,$$

where K is the one dressing both read (section 2.3) and $1/e^2 = 1/(4\pi\alpha)$ in the usual notation. Read on one side only the counts would be 2 and 1, and with three senses 18 and 9: light would then miss by thousands of its errors and colour by hundreds. The pair reading was first found with the measured couplings in view (the measured couplings over K are 8.0012 and 4.0026), and it is what an entry is: one comparison of its two sides, the debit read against the credit, priced once. Light and colour share it and both confirm it, and on a posting's golden entry it also gives the matching's unit its number (section 2.5). On a bond of the carrier the same formula reads the bond's ratio squared, in the carrier's weight and in light's read alike; that only relabels the carrier's variable by two, and it leaves K unchanged. On one golden entry it gives exactly one half (section 2.5).

The reading can be said in the record's own terms. A hop's cost compares what the body reads with what the rest of the record reads. The record is neutral: the turns of an account's five modes multiply to one, so a body and the rest of the record read inverse turns, and their comparison is the body's turn squared, which is the pair. With nothing on the credit side the same comparison reads the body's own turn, which is the one-sided reading. With k of a body's three partners on the credit side the number of senses is $((3+k)/3)^2$: 1, 16/9, 25/9 or 4. Four is the most a neutral record allows, and light asks exactly that (4.0006 ± 0.0004; with two partners light would be 3,390 errors low). Among costs that depend on two kept records only through their comparison, the pair is the one calibrated at balance; $J(\sqrt{d/c})$ is as free of a common change of reference and is the one-sided reading. The credit side is the whole rest of the record because nothing less balances the entry: a part of the record balances a body on every neutral link exactly when it is the rest of the record, and of the counts above only four is a double entry. The entry is priced once because it is one thing: its debit and its credit are its two sides, not two things joined, and asking whether they are one event or two adds a split the entry does not have. Priced as two events, the sum applied inside one entry, light would be 5,547 errors low; priced as nothing, light would have no stiffness at all. The record's own sums, the cost of a ledger, of its first tick and of a double entry at a strain, add postings, and that posting-level sum is the matching's unit (section 2.5). If the cost of a record obeys the record's law of composition, a premise, an entry's two sides held in one record cost either nothing, taken together at balance, or $J(z^2)$, one taken against the other, and never the sum $J(z)+J(1/z)$; so a link that is stiff at all has the count four. Light and colour measure it, and the end of the logarithms takes the same price (section 2.5); reading one side, each side against balance, or the product misses by thousands of errors.

The costs that give these counts do not see every part of a link. Light's pair does not see a half turn of light's link: $1-\cos 2\theta$ vanishes at $\theta=\pi$. Colour's cost does not see the overall turn of the partners' link at all, so turning that link by any sixth root of one costs nothing: light's link, its determinant, turns by at most a half turn. If the ledger summed over these unseen turns, a history would survive only if, on every link, the unseen turns picked up by all the bodies crossing it cancelled, and neither a free electron nor a free quark would exist. So a record holds each link's unseen part. That is a second reading; the quarks' thirds need it whatever form the count takes.

Carried to the cell (section 3), the measured couplings give

$$\frac{1}{e^2}-\frac{2}{g_3^2}=-0.0042\pm 0.0099,$$

which is colour's landing said again, not an independent check; what it adds is that the factor two, the trace, holds at the cell fixed on gravity with no dressing in it.

2.3 The dressing

The carrier is one real number on every event, the logarithm θ of its ratio. A bond b between two events carries the difference $\ell_b=\theta_x-\theta_y$, and the cost of a bond is the ledger's cost of a comparison, $J(e^{\ell})=\cosh\ell-1$ [20]. The carrier sits at weight

$$P[\theta]\propto\exp\Big(-\sum_b\big(\cosh\ell_b-1\big)\Big)$$

with coefficient one (section 2.4). Because the weight depends only on differences across bonds, the identity $\sum_x \langle \theta_x \, \partial S/\partial \theta_x \rangle = N - 1$ for a lattice of N events becomes, bond by bond,

$$\langle \ell_b \sinh \ell_b \rangle = \frac{N-1}{12N} \;\longrightarrow\; \frac{1}{12},$$

the twelve being the number of bonds per event; the bonds are alike, so the identity holds for each. That fixes how much each bond fluctuates. If the fluctuation of a bond were exactly Gaussian with variance s, the identity would read $s e^{s/2} = 1/12$, so $s = 0.0800633$, and the mean loop weight would be $\langle \cosh \ell \rangle = e^{s/2} = 1.040844$. Computed directly on the lattice it is

$$K_m = \langle \cosh \ell_b \rangle = 1.0408495 \pm 0.0000001,$$

a little above the Gaussian value because the bonds have thinner tails than a Gaussian: their fourth moment is 0.7 per cent below $3s^2$. That is the dressing a single loop at one place reads: the mean.

Light is a different field. It lives on the links, and its energy is $\sum_T w_T F_T^2$ over the triangles T of the bonds, where F_T is the field's value around the triangle and w_T is the mean of $\cosh \ell$ over the triangle's three links. Light's stiffness at long wavelengths is the energy per cell of a uniform field strength f, minimized over periodic link fields A:

$$K = \min_A \frac{1}{N} \sum_T w_T \big(f_T + (\operatorname{curl} A)_T \big)^2 \Big/ \frac{1}{N} \sum_T f_T^2 .$$

With every w_T equal to the mean, K would be the mean. Because the weights spread, the field relaxes into the softer triangles and K falls below the mean. On every configuration of the carrier the relaxation is bounded: the minimum lies between the harmonic and the arithmetic mean of the triangle weights. The arithmetic mean is the energy of the field at rest, which is one of the fields tried; the harmonic mean follows from the Cauchy–Schwarz inequality, because a uniform field strength has no divergence on this lattice (the eight triangles on a link carry fluxes that cancel). To second order in the spread,

$$K = \bar{w} - \frac{1}{N \bar{w} \sum_T f_T^2} \sum_q \operatorname{tr} \big(\Pi(q) \, \langle g \, g^\dagger \rangle(q) \big), \qquad g_T = (w_T - \bar{w}) \, f_T,$$

with Π the projector onto the loop values a periodic field can make. With the covariance of the weights taken from a Gaussian carrier of the same bond variance ($s e^{s/2} = 1/12$), the gap on lattices of 4^4, 6^4 and 8^4 cells is 0.000466, 0.000472 and 0.000473, which gives $K = 1.040378$. With the carrier's exact covariance, the same expression evaluated on sampled configurations, the gap is 0.000466 and $K = 1.040384$.

At all orders the minimization is solved exactly. The carrier is sampled from $P[\theta]$ by hybrid Monte Carlo on periodic lattices of L^4 cells, $L = 4$ to 16. On each sampled configuration, for each of the six coordinate planes of the field strength, the periodic link field of least energy is found to a relative residual of 10^{-11}, and the results are carried to infinite volume in $1/L^4$. The gap is 0.000453, and

$$K = 1.040396 \pm 0.000002.$$

An earlier computation gave 1.040399 ± 0.000005. The two were written independently and agree within their errors; the code, the runs and the analysis of the computation used here are

public (see Code and data, at the end). On the samples the identity $\langle \ell \sinh \ell \rangle = (N-1)/(12N)$ holds at every size, and each of the 181,200 minimizations lies inside its two bounds. Of the difference between the Gaussian second order and all orders, 0.0000197, the exact covariance accounts for 0.0000070 and the orders beyond the second for 0.0000127.

2.4 Which stiffness light reads

The carrier has three stiffnesses: the mean loop weight $K_m = 1.0408495$, the long-wave value $K = 1.040396$, and its own stiffness against a slow twist of its own field, 1.040759 ± 0.000002 (an earlier computation gave 1.040747 ± 0.000007). Which one is light's follows from what a coupling is. How a triangle weighs its three links, and whether light changes the carrier it reads, rest on two readings, stated as such below.

The read. A coupling is the coefficient of the squared field strength in a field's law at long waves. For light on the carrier that is the energy per cell of light's field at a uniform loop value f, with light's links obeying their own law in the carrier's weights. On one configuration of the carrier this energy splits exactly into two parts: the cost of f once the links have settled into the weights, and the cost of the field's departure from that settled shape, which does not depend on f. So the coefficient is the relaxed stiffness, the long-wave value, configuration by configuration. The mean loop weight is the cost of a uniform loop value whose links have not settled. It bounds K from above, and it is what one loop at one place reads, not what light's long wave reads.

Not the carrier's own stiffness. For a bond of size u carrying a turn φ, the comparison reads the complex ratio $e^{u+i\varphi}$, and $\mathrm{Re}\cosh(u+i\varphi) = \cosh u \cos\varphi = \cosh u - \cosh u\,(1-\cos\varphi)$. The turn enters as $\cosh u\,(1-\cos\varphi)$: with stiffness $\cosh u$, even in the turn, and never multiplying $\sinh u$, so no current runs between light's turn and the carrier's size. The carrier's own stiffness is its response to a twist of a field that lives on events, and such a twist has loop value zero around every closed triangle. It is never light's.

The weighting, a reading. We read each link as reading the whole of a loop's turn with its own stiffness, so that a loop's cost is the sum of what its links read: the links act in parallel. Given that, a triangle's weight that is unchanged when the lattice's symmetry cycles its three links, and equal to k when every link has stiffness k, is the arithmetic mean of the three. The alternative reading shares the loop's turn among the links in series, each taking a part inversely to its stiffness; it gives the harmonic mean, and light then misses by 17 errors. One link's weight alone breaks the lattice's symmetry, the product of the three is not k when every link has stiffness k, and the geometric mean is not a sum of what the links read.

Light reads the carrier without weighing it, a reading. Light's law depends on the carrier through the triangle weights. If light's turns were summed as weights together with the carrier, the sum over light would leave on each configuration of the carrier a factor that depends on that configuration, and the carrier's law would shift: at first order its coefficient would become about $35/24$, K would be 1.027995, and colour, whose ask does not depend on the hadronic input, would miss by ten of its errors. We read the cell the other way: the links' own turns are not summed as weights there, the same reading that keeps light's own loops out of the matching (section 8). Nothing then multiplies the carrier's weight, and the coupling is the relaxed stiffness averaged over the carrier's own law. What motivates the reading is

that light's turn term, a stiffness times $1-\cos F$, is not a cost that obeys the ledger's law, while a real cost that does obey it gives every turn the weight of the size alone [18]. That motivates the reading; it does not prove it. The same reading carries colour from the cell. The pair does not see a half turn of the partners' link either: for colour's loop, whose turn is a matrix U, the pair reads $\mathrm{Re}\,\mathrm{tr}(1-U^2)$, which vanishes on every turn whose square is one. If the links' own turns were summed as weights at the cell, each triangle weighing $e^{-\mathrm{cost}}$, the pair alone would not keep a triangle near no turn. On the 24 bonds the mean of $\frac{1}{3}\,\mathrm{Re}\,\mathrm{tr}\,U$ over triangles comes out -0.19 at stiffness K a triangle, and -0.29 and -0.32 at twice and four times that: it runs to the half turn's $-\frac{1}{3}$, not to one. Four times the one-sided cost has the same curvature, so the same counts, and gives $+0.89$, $+0.94$ and $+0.97$. The counts cannot tell the two apart. So colour's carry from the cell, which takes the links' turns as small, rests on this reading and not on what the pair costs at large turns.

Computed alternatives miss light on the carry of section 3: one link's weight by 5.7 to 7.4 errors, the geometric mean by 9.6, the harmonic mean by 17, the product by about nine hundred. The read and both readings were identified with the landing known. What the read adds is that it follows from what a coupling is; what each reading adds is a reason that does not mention a coupling, and an alternative that misses.

The coefficient. One identification is left under the read: each comparison weighs its history by e^{-J}, with J the comparison's own cost, so the cost acts as a weight at coefficient one. The cost's unit is not a setting: changing it relabels the carrier's variable, and the dressing does not see a relabelling. The one discrete alternative, the cost placed in the amplitude instead of the weight, would put light nearly three hundred errors low and colour sixteen. Light locates the coefficient at 0.9960 ± 0.0023; that is light's landing said for the coefficient, not a second confirmation of it. The matching's unit rests on the same identification, and through it the combination of light and colour in which the dressing cancels, whose error is mostly colour's, locates the coefficient at 0.964 ± 0.085.

2.5 Matching the lattice to the continuum

A stiffness on a lattice and a coupling in the continuum are compared through a matching, which in general has a scale and a constant. If the ledger's lattice were a regulator, with the fields' own fluctuations summed along its bonds, the constant would be large: the links' own loops summed at the cell take 2.54 from light and 1.52 from colour, and matter summed along the bonds adds 0.12 to light. Each moves light by hundreds to thousands of its errors, and the combination of light and colour that contains no dressing, measured at -0.0042 ± 0.0099, would stand at $+0.50$ with the links' loops summed. Read as a regulator, the lattice is excluded by measurement.

The reading adopted here counts the histories that leave no record differently. Such a history has no count on the record, and its count is read as summed: every whole number of units from one, each weighing one over itself. If $q(t)=\sum_i w_i e^{-T_i t}$ is the return weight of one unit, at $t=k^2+m^2$ for a wave number k and a rest m, the sum is $\sum_{n\ge 1} q^n/n = -\ln(1-q)$, and for small t

$$0 \;\le\; -\ln\big(1-q(t)\big) - \ln\frac{1}{T_1 t} \;\le\; \frac{2M_2 t}{T_1}, \qquad T_1=\sum_i w_i T_i, \quad M_2=\sum_i w_i T_i^2.$$

The one over the count is not put in by hand: marking one recognition of a closed history opens it, so the n markings of a closed history of n recognitions are n open histories, and the closed history weighs one n-th of the open one. The same weight is the logarithm of the number of whole occupations of a mode. So a field adds its coefficient times one logarithm and nothing else, whatever the shape of one unit, and the same for every spin, because the count does not see how a history's length is divided between its stretches: the orbital and the spin parts of every field's coefficient are multiplied alike, where a cut on each stretch would weigh them differently. At first order the comparison has no constant, and a constant common to every field is the same thing as a different unit: the constant and the scale are one number, the proper time T_1 of one unit, and the logarithms end at $1/\sqrt{T_1}$. In the language of the field-theory literature this is the dimensional-reduction scheme with every threshold a pure logarithm; it differs from minimal subtraction by the constant $C_A/(48\pi^2)$ in each $1/g^2$, where C_A is 2 for the weak group and 3 for colour [22]. That is the matching at first order in the couplings, and the comparison is made at that order. At second order a link is exchanged inside a closed history, and the count must say how the stretches between the places where the link meets the history are weighed. The first order has already said it for the link that is read: the count sees the history's whole length and not how it is divided, which is what gives every spin the same end, and the three couplings fix that end to within 0.0078 of an e-fold. Counted instead in whole recognitions, the stretches would move the end of the spin-one-half loops by 0.60 of an e-fold (by 0.15 in half recognitions), whether or not a stretch of no length is allowed. We take an exchanged link to meet a closed history in the same way as the link that is read: one field, one way of meeting. This is a working reading, not a theorem. The statement it extends speaks of the link without saying which, and the first order that fixes it was settled before the second order was computed, so it is not chosen with light in view. Under it, the rules that count each cycle or each stretch of the history in whole units are excluded. It leaves one question open: how the exchanged link's own stretch is counted, within the history's one whole count, by a whole count of its own, or not at all.

Which stretch counts how follows from what a history is on the record. A history is one thing's: a body's history at rest is its own count of recognitions, and a history with no record inside it is summed over its count, every whole number from one, a line plainly and a loop at one over its count. A body's loop with a link exchanged inside it is therefore two histories, not one. The body's loop keeps its own whole count, at one over the count, with the exchanged link's two ends anywhere along it, as the ends of the link that is read are. The exchanged link's stretch is a line of the link's own quantum, which the record counts like every body (its closed histories already enter the first order so); it has no record inside, so it is summed over its own count, plainly, from one. Two other counts are possible in form and are not taken. Counting the link's stretch into the body's one count would need one count over the recognitions of two things, and the record has no history of two things; giving the link's stretch no count of its own would leave a thing's line uncounted. Each also fails when computed: where the link is short and its two ends are close, each leaves a corner that no count reaches, and its logarithm diverges. The count taken here is the only one of the three that is complete. It uses nothing beyond the counting the first order already rests on, and one working reading: that an exchanged link meets a history as the link that is read does.

With this count the second order can be worked out wherever the exchanged link is another thing than the body and the link that is read meets the body only: the closed histories of the

quarks and the leptons with a colour, a weak or a hypercharge link inside. The exchanged link's line, summed over its whole counts, is $\lambda^2/(e^{\lambda^2 k^2}-1)$ in place of $1/k^2$. The weight of a spinor's closed history with the link inside is the published one in a constant field, taken before the link's proper time is integrated [56]. With a body's mass m used only to cut the long histories, that closed history ends at $\ln(1/\lambda^2 m^2) - 2.019$ and the body's own line with the link on it at $\ln(1/\lambda^2 m^2) + \frac{1}{2}$. The two enter as twice the second less the first, $\ln(1/\lambda^2 m^2) + 3.019$, where minimal subtraction has $\ln(\mu^2/m^2) + \frac{15}{4}$; half the difference of the constants is the shift, so the second-order logarithm of this part ends 0.365 e-folds below $\hbar c/\lambda$. This part is 107 per cent of light's second-order slope at the cell. It raises light's ask by 0.00035 and lowers $1/\alpha$ at rest by 0.0045. The histories in which the link that is read meets the exchanged link's own line are now worked out. Their weight has two parts. The first is the closed history with the link inside and the read link on both, at fixed proper times; summed the standard way it gives three tenths of the published coefficient. The second lies where the exchanged link's stretch is so long that the read field bends the link's own quanta: there the closed history is a point on the link's line and enters through its first-order value, and the link's line closes on itself. It gives the other seven tenths, and a weight written at fixed proper times cannot carry it. Carried to any number of dimensions, the two together are the published two-loop expression in every dimension [62]. Under the count the first part ends at $\ln(1/\lambda^2 m^2) - 1.495$ and the second adds no constant, because it lives at stretches far above one unit and its other factor is the closed history's first-order logarithm; minimal subtraction has $-\frac{8}{15}$ on the whole. So this part's logarithm ends 0.042 e-folds above $\hbar c/\lambda$ on spinor rings, and 0.91 e-folds above on the scalar's ring, which carries 1.6 per cent of it for light. It lowers light's ask by 0.000047 and colour's by 0.000049, and raises $1/\alpha$ at rest by 0.0006. The scalar's closed histories with another thing's link inside are worked out the same way: the published scalar weight in every dimension [56], the scalar's own line with the link on it at $\ln(1/\lambda^2 m^2) + \frac{3}{2}$ where minimal subtraction has $\frac{7}{3}$, and an end 0.408 e-folds below $\hbar c/\lambda$; light's ask rises by 0.000033 and $1/\alpha$ falls by 0.0004. One small part is not worked out, the top's closed history with the scalar inside. For it we keep the allowance of version 1.7, a constant of 0.50 e-folds as one standard error: 0.00003 in light's ask and 0.0003 in $1/\alpha$. The largest constant found since is 0.91 e-folds, on the scalar's ring; with that allowance the line would be 0.00005 and 0.0006, and the error of $1/\alpha$ 0.0095 in place of 0.0094. With the line carried below for the links' own quanta, the second order's lines come to 0.00049 in light's ask and 0.0061 in $1/\alpha$, 0.0094 in all with the rest of the error, and 0.0002 in the strong coupling that makes colour exactly $4K$.

The count of the bodies' histories does not settle one kind of history, the one in which the links' own quanta meet one another. Three stretches of one field's quanta join at two places, or two closed stretches touch at one, and none of them is another thing's. The record names none of them the line, but it fixes two things the count must reproduce. The first order has the links' own quanta run their logarithm the whole way to the cell, as the couplings confirm, so the count must give them no mass. And over long histories it must give the couplings' published two-loop running, whose coefficient from the gauge field's own quanta is $\frac{34}{3}C_A^2$ [58, 59]. A count that meets both takes the longest stretch as the line, summed plainly over its count from one; the others close the loop, at one over its count, with the places where the line leaves it anywhere along it. Where a stretch of the loop shrinks to nothing, so that two meeting places become one, the term this leaves is counted with the loop, inside its own count, never through the line. Then the mass vanishes in any number of dimensions, not only

in four, and the long histories give $\frac{34}{3}C_A^2$ exactly. The record does not state this count as a rule. It would if it said that, where a history of one field's quanta has two cycles, the loop counted at one over its count is the shortest cycle and the remaining stretch is the line. We do not assume that; the count rests on the two facts.

The weight is the published one for the gauge field's own two-loop history in a constant field, with its ghosts, expanded to second order in the field [60, 61]; we derived its graphs again from the action in four to eight dimensions, and they agree term by term. Recomputing one pole term of that weight, we find $+1/(12\epsilon)$, from an integral of exactly $1/6$, where [60] prints $-1/(12\epsilon)$; the paper's own total needs the + sign, and [61] prints it. With the loop's total proper time cut at $1/\mu_0^2$ on both sides, the count's logarithm is $\ln(1/\lambda^2\mu_0^2) + 0.544$, where minimal subtraction has $\ln(\mu^2/\mu_0^2) - 0.095$. Half the difference of the constants is the shift, so this part's second-order logarithm ends 0.32 e-folds above $\hbar c/\lambda$. It raises light's ask by 0.00030 and colour's by 0.00063, moving light's ask further from light's value at the cell.

Here the count leaves one thing open, and it is not small. Stretches are whole numbers of recognitions, so the smallest histories, which carry most of this constant, sit exactly where the line is as long as the loop, or as long as half of it, and there the weight jumps. Moving a stretch's own term within the loop leaves terms at the ends of its range: where the line is longer than the loop, at the place where a stretch of the loop shrinks to nothing; where it is shorter, at the place where a stretch of the loop grows as long as the line. When the line is exactly as long as the loop, both kinds of end term fall on the same histories, and the count does not say how they are counted there, nor which of two equal stretches is then the line. Neither fact decides it: the running comes from long histories and the mass from histories with two stretches short against a long third, and the record says nothing on it. Counting each history once, in the ways the count allows, puts this part's constant between -0.20 and $+0.77$ e-folds; the mean of the two sides of each jump, a convention, gives $+0.32$. We quote $+0.32$ and carry 0.52 e-folds, enough to cover every such reading, as a line in the error: 0.00048 in light's ask and 0.0061 in $1/\alpha$. A statement saying how a history is counted where its line is exactly as long as its loop, or as a stretch of it, would settle this.

The second order's constants in one place. Each part's second-order logarithm ends c e-folds above $\hbar c/\lambda$, or below it where c is negative, and c is half the difference of the part's two constants, the count's less minimal subtraction's. An end that lies c e-folds above moves light's ask by the part's slope times c. The top's closed history with the scalar inside is not among the five parts: it is carried in the error only. Light's ask is then its first-order value plus the five shifts, $8.32381334 + 0.00063722 = 8.32445056$, printed above as 8.324451, and $1/\alpha$ at rest is the measured value less light's distance from $8K$ over the slope of section 3: $137.035999 - (8.324451 - 8.323168)/0.0794553 = 137.0199$. The second and third columns are the constants under the count and under minimal subtraction. Slopes are light's, for each e-fold of the end, and shifts are of light's ask; both are in units of 10^{-6}. The count's constants are built as follows. Spinor histories with another thing's link inside: twice the body's own line, $\frac{1}{2}$, less the closed history, -2.01927. The read link on the exchanged link's own line: three tenths of -1.49476 on spinor rings; on the scalar's ring the part at fixed proper times is $-\frac{1}{4}$ of a whole of $\frac{1}{3}$ and ends at $+0.24256$, so the constant is three times that. Scalar histories with another thing's link inside: the closed history gives $\frac{3}{2}\ln(1/\lambda^2 m^2) - 1.88157$ and the scalar's

part	count	subtraction	c	slope	shift
spinor histories, another thing's link inside	+3.019	$\frac{15}{4}$	−0.36536	−969.00	+354.03
the read link on the exchanged link's own line, spinor rings	−0.448	$-\frac{8}{15}$	+0.04245	−823.52	−34.96
the same on the scalar's ring	+0.728	$-\frac{13}{12}$	+0.90551	−13.73	−12.43
scalar histories, another thing's link inside	−0.566	$\frac{1}{4}$	−0.40789	−79.86	+32.57
the links' own quanta	+0.544	−0.095	+0.3193	+933.33	+298.01
all five					+637.22

own line with the link on it $\frac{1}{2}\,[\ln(1/\lambda^2 m^2) + \frac{3}{2}]$, together $2\,[\ln(1/\lambda^2 m^2) - 0.56579]$, where minimal subtraction has $2\,[\ln(\mu^2/m^2) + \frac{1}{4}]$, from $-\frac{2}{3}$ and $\frac{1}{2}\cdot\frac{7}{3}$. The links' own quanta: $\frac{8}{17}$ of 1.15534.

One unit is one recognition: each of the eight columns, four accounts with a debit and a credit, posted once. The four debits are an orthonormal frame of bonds, and so are the four credits, so one recognition's second moment is exactly twice the round one and its proper time is exactly λ^2: the logarithms end at $\hbar c/\lambda$. Each posting weighs a displacement of x entries by $e^{-x^2/2}$ on each account. That number has been reached in three ways, which are one number and not three supports. It is the least state of an action that weighs a count and its turn alike, the form the ledger's posting law has; the function is its own transform, with the turn in radians. It is the cost's own curvature at balance. And it is the pair cost of one golden entry: read on its pair a comparison z costs $(z - 1/z)^2/2$, half the squared difference of its two sides, and the golden entry's sides differ by exactly one. The principle is older than the comparison at the cell, and so is the number. The record prices a delta of Δ postings on one side of one account at $\cosh(q\Delta) - 1$, with q the strain one posting presents, and had left q open: it names values for it, each fixed by one definition, and derives none. Among them is the half adopted here: where a debit posting with a credit posting costs one, at $q = 2\ln\phi$ with $\phi = (1+\sqrt{5})/2$, one posting presents the ratio ϕ^2 and costs $J(\phi^2) = \frac{1}{2}$. What was chosen with the end in view is that a displacement of one entry in a history that leaves no record is costed as that posting. The couplings then decide what the record had left open. Read back from the three couplings, the strain one posting presents is 0.9646 ± 0.0069, against $2\ln\phi = 0.9624$. The record's own convention, $q = 1$, at which one post costs $\cosh 1 - 1 = 0.543$, would end the logarithms at $1.042\,\hbar c/\lambda$, 1.7 errors from the end light and colour ask and 5.0 from the end the three couplings ask. The values it orders beside these two, one golden rung, one bit and one coin a posting, miss by 44 errors or more. A debit post with a credit post at one golden rung each, $2J(\phi) = 0.236$, taken for one entry's cost, would end them at $0.69\,\hbar c/\lambda$: 49 errors off, and more than eight in the combination of light and colour from which the dressing and the count's four both drop out. With the turn counted in whole cycles the same principle would end the logarithms at $2.51\,\hbar c/\lambda$, and with the square of the least state at $1.41\,\hbar c/\lambda$; the measured end excludes both, by more than thirty errors. The whole-cycle form is the posting law's own: the law states no kernel, and its least state at fixed norm, worked out here, is $e^{-\pi x^2}$ on each coordinate, repeated around the law's three phases, with x in the law's own

extent; the kernel adopted here is the same function with the bond at $1/\sqrt{2\pi}$ of that extent. The radian, the bond as the posting's extent and the least state itself were chosen with the end in view. Read back from the measured couplings, one entry costs 0.519 ± 0.013; the entry read on one side only (0.118) is 30 errors off, and four times that cost at small size (0.472) 3.5.

The measured couplings put the end at $(0.98 \pm 0.04)\,\hbar c/\lambda$ from light less twice colour, which contains no dressing, and at $(1.019 \pm 0.013)\,\hbar c/\lambda$ from light and colour with the dressing: 7.70 ± 0.20 postings of bond size to a unit, where the ledger has eight, and a unit proper time of 0.963 ± 0.025 in units of λ^2. Seven is 3.7 errors away and nine 6.0. The weak share hardly involves the dressing, and given the form of the stay (section 5) it reads the end by itself: $(0.996 \pm 0.009)\,\hbar c/\lambda$. The three couplings together put the end at $(1.002 \pm 0.008)\,\hbar c/\lambda$: a unit of 0.995 ± 0.015 in units of λ^2, 7.96 ± 0.12 postings, with seven and nine more than seven errors away, and one entry's cost at 0.502 ± 0.008. Left free, the dressing is fixed by the same three couplings at 1.040576 ± 0.000100; the computed 1.040396 lies 1.8 of that error below. The coefficient of section 2.4 states the same shortfall: with the end held at the cell, the three couplings locate it at 0.9962 ± 0.0022, 1.7 errors below one. With no determination of the hadronic input used at all, the couplings still put the unit at 1.059 ± 0.040 and ask the input 0.02806 ± 0.00012. These figures are the three landings read as an end, not a further test of the closure, and the share's part of them stands or falls with the form of the stay. One neighbour that is not a whole number of postings, the exact cost read as an amplitude (an end at $1.0305\,\hbar c/\lambda$), lands on light and colour, so those two fix the unit to two per cent, not to the integer by themselves; with the weak share in, it does not land ($\chi^2 = 16.1$ for three). That neighbour, and the one that weighs a posting by the exact cost of its length as a weight (an end at $1.290\,\hbar c/\lambda$), are excluded without a measurement: a step weighed by the exact cost of its length obeys no law of comparison in a plane, whatever product term it carries, so only the plain law weighs where a posting goes. The first also puts the cost in the amplitude, which the carrier excludes (section 2.4). Minimal subtraction at the cell misses light by more than seven errors, and reading the coupling at the zone's edge π/λ by more than seventy. A cut on each line of a loop, which is what a regulator of fields does, would give scalars, spinors and vectors different ends; the measured couplings ask for one end for every spin, within 1.8 of the error, and the cut lies 21 errors away. That 1.8 is light's shortfall, seen in the one combination of the three couplings that does not involve the end. The inside of a history that leaves no record is read as not on the record's places: lattice places inside such a history would be refused by gravity's two bounds, the speed of its waves and the absence of a mass for the graviton, and no coupling enters that.

Gravity at the cell belongs to this matching. In units of the cell's area Newton's constant is $1/(4\ln 2) = 0.36$, which is not small. If the strain's own histories were summed like a field's, light would move by tens of per cent (by a factor 1.41 in a cutoff scheme [23]). Within two errors the closure leaves room for a relative addition between -0.00003 and $+0.00033$. The result therefore rests on the fourth of the statements listed below: the strain is the record's own geometry, sourced and never summed.

Four statements carry the matching and are not yet derived from the ledger's foundation: that the count of an unrecorded history is summed, that a posting which leaves no record is in the least state of the equal-weight action (which sets the unit), that the links' own turns are not summed as weights at the cell (section 2.4), and that gravity's strain is never summed. With them the matching has no free number.

3 The carry from rest to the cell

What the measured couplings are at the cell is a Standard Model calculation, and we use it as it stands. The state of the Standard Model at 200 GeV is taken from the interpolation formulas of Alam and Martin [28], which match the on-shell measured inputs (the fine-structure constant, the Fermi constant, the masses of the Z, the top quark and the Higgs boson, the strong coupling, the quark and lepton masses, and $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$) to the Lagrangian couplings at three loops and beyond. From there the three-loop renormalization-group equations [31, 32, 33] run every coupling to the cell. Three effects beyond that running are known and are applied to light's ask: the complete three-loop running of the Yukawa couplings and the Higgs self-coupling, as the reference code's own published running shows from a shared start (+0.000005; [29]); the four-loop gauge terms (+0.00005; [34]); and the top quark's decoupling from light at order $\alpha\alpha_s^2$, which the two-loop matching at the Z [30] leaves out (−0.00006; [36]). Converted to pure logarithms, with the 2026 electroweak inputs, $\alpha_s(M_Z) = 0.1180(9)$ and the 2026 lattice value of the hadronic input, the measured couplings ask at the cell:

	asked at the cell	error
light, $1/e^2$	8.324451	0.000750
colour, $1/g_3^2$	4.164320	0.005078
weak share, g^{-2}/e^{-2}	0.4675136	0.0000661

These are the asks with the matching's second order as far as it is computed. At first order they are 8.323813 ± 0.000572, 4.163585 ± 0.004975 and 0.4674982 ± 0.0000583; the computed part of the second order moves them by +0.000637, +0.000736 and +0.0000153, and the lines for the part not computed and for what the count leaves open add 0.000485, 0.001018 and 0.0000310 to the errors in quadrature (section 2.5). Those lines move the three together: with them the correlations are +0.38 between light and colour, −0.11 between light and the share and +0.02 between colour and the share.

Light's error has these parts: the hadronic input (0.000534), the strong coupling (0.000195), the top mass (0.000058), and, together 0.00004, the matching of the reference state (its last revision moved light by 0.000015, its interpolation formulas by at most 0.000017), what remains of the matching at the Z after its three-loop top term (0.00003), and the running beyond four loops (0.00001). Kept as version 1.0 kept it, with the revision of the reference state added in quadrature over the two couplings (0.000573), light's carry error would be 0.000810 and, with the matching's line for its uncomputed second order (section 2.5), light would sit at −1.36 instead of −1.71, with $\alpha^{-1} = 137.0199 \pm 0.0119$ and the input asked 0.027939 ± 0.000087; the value does not move and the conclusion does not depend on the choice. We use the signed step because the revision changed the electroweak scale, which moves g and g' in opposite directions (by −7 and +2 parts in 10^5) and leaves light, which is fixed by α and the hadronic input, almost unchanged (+0.000015), as a change of the Z mass does (0.000004); the matching of light itself, which both versions share, is carried separately (0.00003). The weak share's error keeps that revision whole, because a change of the electroweak scale moves the split even when it leaves the sum alone. Colour's error is almost entirely the strong coupling's. The series is short: from one loop to two the running moves light by −0.0531, from two to three by −0.0015, and the four-loop terms by +0.00005, opposite in sign and a thirtieth of the

three-loop term [35]. A series whose next term is computed, and is a thirtieth of the last, does not have the last term as its error.

The running contains no gravitational term. In a mass-independent scheme such as this one, Newton's constant can enter the running of a dimensionless coupling only multiplied by a squared mass of the theory ($Gm_t^2 \approx 2\times10^{-34}$) or by the cosmological term [25]; over the thirty-nine e-folds from the Z to the cell that moves light by less than 10^{-30}. With a momentum cutoff in a particular gauge, gravity's power-law correction would make light's stiffness 1.41 times larger at the cell [23]; it depends on the gauge [24], it is absent in dimensional regularization [25], it does not act as a universal running coupling [26], and in an asymptotically safe completion it would add about 0.18 [27]. Those effects are the size of the higher-dimension terms at the cell. They belong to the matching (section 2.5), which compares the coefficients of the dimension-four term, not to the carry.

The carry assumes that no field carrying electric, weak or colour charge exists between the Z and the cell other than the Standard Model's. That is a premise, and with the closure it becomes a prediction. Every such field softens light at the cell with the same sign. One charged fermion at 1 TeV would move light by about eight hundred of its errors, and one anywhere below 0.957 of the cell's energy (7.01×10^{18} GeV) would move it by more than one error. Heavy neutral fields count too when they bind strongly to the lepton doublets: within one error, the sum over them of $y^2\ln(E/M)$ stays below about 9.

The value at rest follows from the value at the cell through one slope. The fine-structure constant enters the carry only through the coupling at the Z, $1/\alpha(M_Z) = (1-\Delta\alpha)/\alpha$. Every vacuum term in $\Delta\alpha$ is itself proportional to α, so $\Delta\alpha/\alpha$ does not move with α, and a change in $1/\alpha$ moves $1/\alpha(M_Z)$ one for one. At fixed α the hadronic input moves $1/\alpha(M_Z)$ by $-1/\alpha$ per unit and light at the cell by -10.888 per unit. So

$$\frac{d(1/e^2)_{\text{cell}}}{d(1/\alpha)} = \frac{10.888}{137.036} = 0.0794553,$$

close to the one-loop value $1/(4\pi) = 0.0795775$.

4 The fine-structure constant

With $K = 1.040396$, light at the cell is $8K = 8.323168$, against 8.324451 ± 0.000750 asked with the matching's computed second order and the line for its uncomputed parts (section 2.5). With the dressing's own error added, that is -1.71 of the error. Through the slope,

$$\boxed{\alpha^{-1} = 137.0199 \pm 0.0094}$$

against the measured 137.035999177(21). No continuous number in it was fitted to light: K is computed, the count is whole, the cell is fixed on gravity, and the carry is the Standard Model's own. With the earlier computation of the dressing, $K = 1.040399$, it is 137.0202 (-1.68). If the 2026 lattice determination's total moved by the 0.00007 our recomputation finds, the result would move up by 0.0096, to 137.0294, and light would stand 0.69 of its error below its ask; the paper keeps the value as printed.

When every input of the carry that the ledger has its own number for is taken from the ledger instead of from measurement (the strong coupling solved so that colour at the cell is exactly

$4K$, which gives $\alpha_s(M_Z) = 0.11849$, and the quark masses of the ledger's mass table, with the top quark at 172.46 ± 0.37 GeV), light asks 8.324343 ± 0.000726 at the cell and

$$\alpha^{-1} = 137.0212 \pm 0.0091,$$

-1.62 of the error. The ledger's values move the result toward the measurement.

read of the carrier	K	earlier	light	α^{-1}	light's read?
long wave, all orders	1.040396	1.040399	−1.71	137.0199	yes
second order, own covariance	1.040384		−1.83	137.0187	approximation
second order, Gaussian	1.040377	1.040378	−1.91	137.0179	approximation
the carrier's own stiffness	1.040759	1.040747	+2.16	137.0565	no (§2.4)
mean loop weight	1.0408495	1.040851	+3.13	137.0655	no (§2.4)

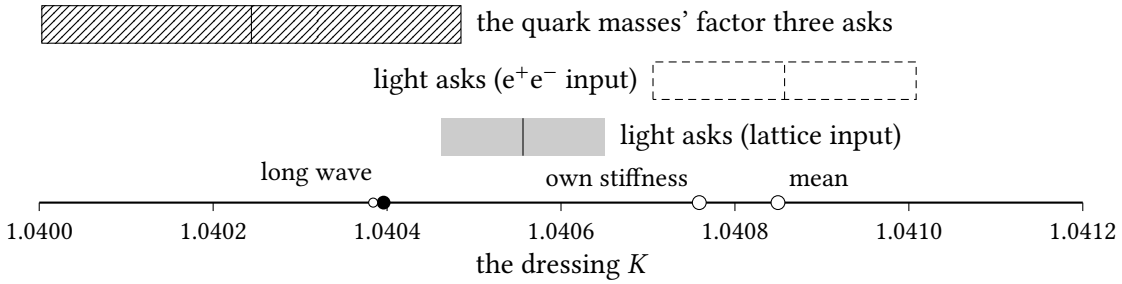


Figure 1. The dressing each measurement asks for, against the carrier's computed reads. Filled dot: the long-wave value, light's read; small circle: its second-order approximation; open circles: the carrier's own stiffness and its mean, which are not light's (section 2.4). Light's ask on the lattice hadronic input (grey) lies 1.71 of its error from the long-wave value. The quark masses' factor of three (hatched) asks a dressing within one error of it. On the data-driven hadronic input (dashed) light's ask moves up by 0.0003: there the own stiffness and the mean would land and the long-wave value would not, but neither is a read light can take.

4.1 What the agreement weighs

The result rests on discrete choices, and they can be counted. Ten were written down, each with its natural alternatives and equal weights, before any combination beyond the eighteen on record had been scored; the reported combination's agreement was already known when they were chosen: light's count (8, 2, 4, 16); colour's (4, 2, 8, 16); which stiffness of the carrier light reads (three); how a triangle weighs its links (five); the weight's coefficient (1, $\frac{1}{2}$, 2); the bonds per event (12, 4, 8); the matching constant (none, minimal subtraction, and three lattice-type constants); the reading scale (six multiples of $\hbar c/\lambda$); the form of the weak share (seven); and the dressing the share reads (three). That is 1,360,800 combinations, each scored on the carry of section 3. None lands light, colour and the weak share within one error. The reported combination lands all three within two (-1.71, -0.54, -0.50).

The search for where the logarithms end was wider than any of these menus. Thirty-two forms for the end were tried while it was sought: seven cycle lengths and three walks on the bonds, four ways to weigh a step by its interval, and eighteen closed forms found by hand;

more than half put the end between 0.96 and 1.05 of $\hbar c/\lambda$. Before any of them the record already held values for the cost of a posting, none of them derived, some from an earlier attempt on the same constant. Read as one entry's cost, one is the half adopted here: one posting where a debit posting with a credit posting costs one. A cost of $\ln\phi$ puts the end at $0.981\,\hbar c/\lambda$ ($\chi^2 = 11.1$ for three, against 3.4). The record's own convention, one post at strain one, puts it at $2\sinh\frac{1}{2} = 1.0422$, 5.0 errors from the three couplings. One value the couplings cannot tell from the half: $J(9) - J(8) = 71/144$, what one more posting adds to the cost of a count of eight; it is the half less $1/144$ ($\chi^2 = 4.8$). Every other value found there misses by more than eight errors. The earlier attempt has the number one half too, as the least cost of a whole-number flow of the field a posting sources: one flux quantum on the bond that joins the posting's debit to its credit. That is half the square of one unit, as the adopted half is, but in that attempt's own units, and there it is a field's cost. The attempt keeps the cost of the posting itself as a separate number, with candidates of its own. The unit's number does not come from that half. Four of the thirty-two fit the couplings better than the reported package, all of them walks on the bonds: cycle of 4 postings (+0.64, −0.39, −0.68); walk on the 24 bonds, in pairs (+0.64, −0.39, −0.68); walk of 16 postings, in pairs (+0.71, −0.22, +0.15); cycles of three postings, which end the spinors' logarithms at the cell and the vectors' 0.3 per cent below it, put light, colour and the share at −1.34, −0.46 and −0.23. It is excluded without using the couplings, twice over: three single postings never close, since an odd number of half-integer bonds cannot sum to zero (a closed walk of three steps is a triangle, which takes a unit bond, that is two postings); and it is a walk on the bonds, which gravity's two bounds refuse (section 2.5). Twelve of the thirty-two go that way, with no coupling used: two set by the exact cost of a length, which a theorem excludes, and the ten walks on the bonds. Of the twenty left, the couplings keep the end at the cell.

What the agreement weighs is how often a menu this large would produce a fit this good by accident. Let the three couplings lie anywhere within one per cent of their values, uniformly and independently, and ask for the chance that some combination fits them at least as well as the reported one does, by the sum of the squared pulls; with the correlated chi-square as the measure instead, the chances given below as 0.31, 0.041 and 0.00005 would be 0.29, 0.038 and 0.000047. Table 1 gives it for each menu, without and with the thirty-two forms for the end; the record's own convention for one post, $\cosh 1 - 1$, is counted in both.

Three numbers say what the agreement weighs, each without and with the search for the end. On the menus fixed before the count, the chance of an accidental fit this good is 0.022, and 0.041 with the search (0.0065 on the ten-choice menu). That null is generous to the result: one per cent is much wider than the spread of what the candidates predict. Conditioned instead on the coarse structure, with each coupling anywhere among the values the candidates of the earlier menu predict for it, some candidate fits all three as well as the reported one with chance 0.18, and 0.31 with the search (light and colour together: 0.69 and 0.95; light alone: 1.00 and 1.00). After the derivations that were found once the agreement was known, the first chance falls from 0.0015 to 0.00005 as each is granted, and from 0.0029 to 0.00005 with the twenty forms of the search that survive the exclusions made without the couplings; the last value is the package's own chance, with nothing left to choose. Each derivation rests on premises that do not mention a coupling (sections 2.2 to 2.5), but each was found with the agreement in view, so we give all of these numbers and do not choose between them; a derivation made without the couplings in view is the test of that. For the counts it is settled:

menu	without the 32	with them
fixed before the count, with the reported agreement already known		
the ten choices above	0.0065	
the alternatives each derivation of section 2 was tested against: the counts, the ends of the logarithms, the reads and weightings of the carrier	0.022	0.041
the same, with the hadronic input also a choice	0.042	0.073
wider: counts up to 24, nine scales, forms of the share off the photon line, the hadronic input a choice	0.076	
after the exclusions derived once the agreement was known		
the read, the weighting, the matching constant, the ratio of the counts, the bonds per event, the time bond's dressing (and with it the dressing the share reads) and the ends of the logarithms that break a theorem, derived	0.0015	0.0029
also: a weight acts on odds	0.0012	0.0024
also: the dressing sits on loops, so the share's form is fixed	0.00031	0.00061
also: an entry is priced once, as one comparison of its two sides (the number of senses), and a posting that leaves no record is in the least state (the end of the logarithms)	0.00005	0.00005

Table 1. The chance that some combination of a menu fits light, colour and the weak share at least as well as the reported package, if the three couplings had been anywhere within one per cent of their values; without and with the thirty-two forms tried for the end of the logarithms. The upper rows were fixed before the count and count every form tried; the lower rows remove alternatives by derivations found after the agreement was known, and count the twenty forms that no exclusion without the couplings removes. Light's matching at the cell is taken with the second order computed except the top with the scalar and the count of the links' own quanta at the jumps of their weight; both are carried in each coupling's error.

an entry is one comparison of its two sides and is priced once, so the counts follow, and the other pricings miss, two events by thousands of errors; the measured couplings over K, 8.0012 and 4.0026, confirm it. It was first found with them in view. These chances take light's matching at the cell at first order with the second-order part computed in section 2.5, and carry the parts not yet computed as a line in each coupling's error. On the errors without that line the three would be 0.34, 0.051 and 0.00007: the line makes the package's own fit better (light at -1.71 instead of -2.24), and a better fit is harder to match by accident. They are larger than version 1.7's (0.21, 0.021 and 0.00002) because the computed second order moves light's ask away from the package.

The precision of the agreement carries little beyond the coarse structure and colour. Given that light's coupling at the cell lies among the carrier's stiffnesses times eight at all, some candidate fitting it as well as the reported one does is what chance gives nearly always. What carries the weight is that light and colour sit at whole counts times one stiffness near 1.04 at the cell fixed by gravity, together.

On every menu the couplings themselves select the counts 8 and 4, the coefficient one, the twelve bonds per event and the reading at $\hbar c/\lambda$, and they leave no weight on any lattice-type

matching constant. On the ten-choice menu they lean to pure logarithms (0.94 of the weight) and to the carrier's own stiffness (0.62) over the long-wave value (0.28) that the derivation takes; the derivations do (sections 2.4 and 2.5). They cannot tell the share's forms apart; that is left to the derivation of the stay (section 5). Of the eleven ends of the logarithms in the derivations' menu, two break a theorem (section 2.5): they are set by the exact cost of a length, and the nearest rival, at $1.0305\,\hbar c/\lambda$, is one of them; that cost weighs a displacement by no law of comparison. Apart from the end at the cell, the other eight each break a reading or a measurement and miss light by more than four errors.

When the hadronic input is counted as a choice and the read is left free, another rival fits as well. On the electron–positron input 0.02760(10), the carrier's own stiffness or its mean, with the share's form $(2K + 1)/3$, lands all three couplings within one error. Neither part of it survives what is now derived: the own stiffness and the mean are not reads light can take (section 2.4), and the form $(2K + 1)/3$ leaves the time bond undressed, which the lattice's symmetry forbids (section 5). With those exclusions in place the couplings put between 0.83 and 0.92 of the weight on the lattice input, and 0.99 once the ends of the logarithms that break a theorem are removed as well.

These chances take the coarse agreement as given: that light's and colour's couplings at the cell lie within one per cent of whole counts times one stiffness near 1.04 at all. If both couplings at the cell could have been anywhere between 1 and 64 on a logarithmic scale, that alone happens about once in 1,200 with the carrier's own reads at the cell, and about once in 20 with every choice of the ten-choice menu left free. Taking both levels together, the chance that some combination of that menu lands all three couplings within one error is about two in a million (1.6×10^{-6}).

While the dressing was being sought, 120 other values for it were tried: ratios of the ledger's fixed counts, laws of its draw counts, and a few closed forms; ten of them lie between 1.035 and 1.047. Added as further reads to the ten-choice menu with the coefficient and the bonds held, they raise its chance from 0.0034 to 0.021, and one of them, 1.04115, fits better than the reported package (+0.70, −1.19, −0.28), with no matching constant and the share's form $D = K$, two choices the derivations remove.

The package was fixed before an input fault in the carry's starting state, found for another reason, was corrected, and the correction moved light toward the measurement with nothing chosen again.

5 Colour, the weak share and the quark masses

Colour. With the same K, colour at the cell is $4K = 4.161584$ against 4.164320 ± 0.005078 asked with the matching's second order: -0.54. Equivalently, the strong coupling at the Z that makes colour exactly $4K$ at the cell is $\alpha_s(M_Z) = 0.11849 \pm 0.00019$ in the adopted convention, the error being the line for the second order's uncomputed parts (minimal subtraction at the cell would move it by about 0.001), against the measured 0.1180 ± 0.0009 [15]. Colour's window for K is loose (± 0.0013); it confirms that light and colour read one common dressing more than it tests the dressing's value.

The weak share. Light's stiffness splits into the weak part and the hypercharge part, and the split comes from a count on the record of what reads a link. It reads a link's stiffness as the sum, over every carrier that moves with the link, of that carrier's index, the sum of the squared charges of its states. Matter's three generations enter at a weight w: each generation adds 2 to colour, 2 to the weak link, $10/3$ to hypercharge and $16/3$ to light. The links' own carriers enter at weight one: colour's add 3, the weak link's 2, hypercharge's none, and the charged weak carriers add 2 to light. The scalar doublet enters at a weight w_h, adding $\frac{1}{2}$ to the weak link and to hypercharge and 1 to light. So colour counts $6w + 3$, the weak link $6w + 2 + w_h/2$, hypercharge $10w + w_h/2$, and light $16w + 2 + w_h$, the weak and hypercharge counts together. This count fixes ratios only; light's $8K$ sets the size. Light is twice colour, as the loop law requires at every dressing, exactly when $4w + w_h = 4$. Matter is read at its moving share at balance, $w = 1 - \sigma$, where the stay σ is the share of a body's moves that rest; the line then gives $w_h = 4\sigma$, which we call D. The weak count is $8 - D$ and light's is $18 - 3D$, so

$$\frac{1/g^2}{1/e^2} = \frac{8 - D}{18 - 3D},$$

which is $7/15$ when nothing is dressed ($\sigma = \frac{1}{4}$).

The stay is read from the carrier. A recognition has sixteen moves, one for each pair of an account debited and an account credited: four are rests and twelve are transfers. A rest closes one of light's triangles (its two postings and the time bond); a transfer closes none [18]. The lattice's symmetries carry the time bond onto a posting bond, so the rest's triangle is dressed by the carrier's mean loop weight K_m. Light's turn along an open path can be removed by relabelling the phases at its events, so the carrier has nothing on a transfer to dress. If weights act on odds and a move reads the carrier only through the light it closes, the stay is $K_m/(K_m+3)$ and $D = 4K_m/(K_m + 3)$, which gives the share 0.4674803 against 0.4675136 ± 0.0000661 asked with the matching's second order: -0.50. The reading that weights act on odds was adopted with the share in view. Had a move instead weighed the carrier's own cost on the two links it crosses, the share would move by less than a twentieth of its error. The other stay forms on record each break one of these steps: a quarter of the rest's weight does not act on odds, and a triangle with its time bond undressed is ruled out by the symmetry. Writing $D - 1 = c(K_m - 1)$, the form gives $c = 3/(K_m + 3)$, three quarters when nothing is dressed, because twelve of the sixteen moves are transfers. The measured share allows c from 0.71 to 0.83, so on its own it would also admit $4/5$; no count of the moves gives $4/5$.

Every step from the indices to the stay is a reading, not a derivation, and the form was adopted with the measured share in view. The share hardly depends on the dressing (its slope is 0.02 per unit of K_m), so its landing tests this form, not the value of K. The weak coupling and hypercharge at the cell are light and the share recombined, not new equations.

The rule that gives light $8K$ and colour $4K$, $1/g^2 = 2STK$ for a link read on carriers of index T, does not give the weak share. Applied to the weak link and to hypercharge separately, each read on the scalar doublet, it gives $4K$ each and a share of one half; the measured 0.4675 excludes that by about 490 errors. The two counts are one. Light's total and colour's do not depend on which unit charge reads a loop; the division of light between the weak link and hypercharge does. It is the readers' mean split number p: the weak link has $(1 - p)/2$ of light's stiffness and hypercharge $(1 + p)/2$ (p is the cosine of twice the weak mixing angle at the cell), and p is the mean, over what reads the loop, of each reader's own split number,

weighted by its share of light's stiffness. The weak link's own carriers read -1; a body that holds one of the two columns reads 0, which is the count rule's reader and gives the share one half; matter as a whole, all five modes read alike, reads $\frac{1}{4}$; the both-columns record, a debit and a credit on one account, reads 1. The occupation count above is this mean over its census, $(12 \times \frac{1}{4} - 2 + 0)/15 = 1/15$ undressed: the rule's one half assumed that only single columns read. Within a single cost of the two columns' link, $a\ \mathrm{tr}\, X^2 + b\,(\mathrm{tr}\, X)^2$ for the link's generator X, light's stiffness is $a + b$ and $p = b/(a + b)$; the measured share asks $p = 0.0650 \pm 0.0001$, a cost on the link's overall turn worth 6.5 per cent of light's. The sign settles what the pair's credit side is: a cost that compared a body's debit column with its own credit column would raise the share to one half or above, 492 errors off, so the credit side is the rest of the record (section 2.2). The value of p is not derived. Three forms land it: the occupation count's line with the stay, above, and two given below.

The term that tells hypercharge from the weak link is already in the ledger's cost. The cost of a complex ratio z, $J(z) = (z + 1/z)/2 - 1$, is the mean of its debit reading z and its credit reading $1/z$, less one. On a bond of size u carrying a turn φ the mean is

$$\cosh(u + i\varphi) = \cosh u \cos\varphi \ + \ i \sinh u \sin\varphi.$$

Light's and colour's stiffness read the real part (section 2.4). The imaginary part is odd in the turn, and it is present only as far as the bond's size unbalances the two readings: it is $\tanh u$ times $\cosh u \sin\varphi$, and it vanishes on a bond at balance. On the two columns, turned by φ_1 and φ_2, the imaginary parts add to

$$\sinh u\,(\sin\varphi_1 + \sin\varphi_2) = 2 \sinh u \sin\frac{\varphi_1 + \varphi_2}{2} \cos\frac{\varphi_1 - \varphi_2}{2}.$$

This reads the columns' total turn, which is hypercharge's. It is zero on the weak turn, where the two columns turn oppositely, whatever the size of the turn, and zero on a colour turn that turns two partners oppositely and leaves the third. So this term tells hypercharge's turn from the weak link's. These are identities of the cost; no measurement enters them.

Its size on the carrier is computed. The mean of $\tanh^2 u$ over a bond is $t = 0.0695885 \pm 0.0000011$, from the same sampling that gives the mean loop weight of section 2.3. One form built on t lands the share. Let the columns' total turn cost t where each column's own turn costs one, $a\,(\varphi_1^2 + \varphi_2^2) + a\,t\,(\varphi_1 + \varphi_2)^2$, the two sharing light's $8K$ between them, as weights that act on odds do. Then $b/a = t$ and $p = t/(1 + t) = 0.0650610$, hypercharge's stiffness stands to the weak link's as $1 + 2t$ to one, and the share is

$$\frac{1}{2\,(1 + t)} = 0.4674695$$

against 0.4675136 ± 0.0000661 asked: -0.67. In this form the share's departure from one half is the carrier's doing: with every bond at balance $t = 0$, and the share is the rule's one half. The share then tests the carrier as light does. It puts the coefficient of the cost in the carrier's weight at 1.0017 ± 0.0026, where light puts it at 0.9960 ± 0.0023.

The form is located, not derived: it was found with the measured share in view. Two steps are missing. One is why a record that holds both columns should weigh the carrier's mean squared imbalance, one number on every triangle, where a record that holds one column weighs one. The other is why the weights act on odds here; were the term simply added, light

would be about 770 of its errors high, or colour 54 low. Taken as p itself, t misses by 35 errors. Eighty-eight values have been set against the share, sixty-five of them built on the carrier's bond, and fifteen fall within one error of it. Three of the fifteen have a reading behind them: the occupation count's line with the stay (-0.50); $p = K/16$, an account's own rest, one move in sixteen, read as its both-columns record and dressed by K (-0.39); and this one (-0.67). The other twelve are three variants of the second, one of the first under a move the record does not write, an estimate made by hand, a form tried in earlier work, and six coincidences that were noticed and dropped. The three forms differ by a quarter of the share's error, so the share does not choose among them. They differ in what they say of a ledger with no carrier: $7/15$, $15/32$ and one half.

Each of the three lands with one statement the record does not now make, and none is derived. The carrier's imbalance as odds needs the columns' total turn to cost one number on every triangle, the carrier's mean t: a weight in a count, as the mean loop weight is for the stay, and not the cost of a bond's own turn. If each triangle instead weighed $\tanh^2 u$ of its own bonds, hypercharge's stiffness would be a relaxed one, as light's is (section 2.4): its links settle into the softer triangles and the total turn's cost falls from t by 0.00177, the weak link's weights are the same on every triangle and its stiffness does not fall, and p comes out 0.06351, 11.1 errors low (with the dressing kept inside each triangle's weights, 9.0 high). The record's own law, on a record holding both columns, gives two lawful totals instead, 89 and 36 errors out at long waves. $K/16$ needs an account's own rest weighed against the plain sixteen moves, a reading the record once held and then replaced by the normalized stay, on which the own rest's share is 4.5 errors out. The occupation count's line needs matter read at its stay's value at balance, not at its place. The carrier's imbalance and $K/16$ use no stay, so under them the stay's form enters no coupling. The occupation count's line uses the stay at balance, and there the share itself tests its form: the rests' share lands (-0.50) and the quarter misses by 3.8 errors. This form holds while a transfer, which closes no loop, weighs one; were the carrier read inside each posting with the pair's half ratio, transfers would be dressed, this form would be 9.6 errors out, $K/16$ would be 14 out, and the printed share would rest on the carrier's imbalance alone.

Free fall: the split's number is not a local stay. A form whose number follows matter's stay at its own place would make the weak share depend on gravity. Near a still body the stay rises with the depth of the potential, $\sigma = \sigma_0 + (1 - \sigma_0)B$ with $B = 2|U|/c^2$, so matter's moving share falls as $(1 - \sigma_0)(1 - B)$. Read there, the occupation count's line moves the weak share at the cell by $+0.080\,B$. In ordinary matter the proton's weak charge, $1 - 4\sin^2\theta$, then changes, and with it each nucleus's weak neutral-current energy: about 1.4 eV a nucleon in titanium and 1.7 in platinum (zero-range exchange in a uniform nucleus). Titanium and platinum would fall differently by $\eta = 3.2 \times 10^{-11}$, or a tenth of that if correlations between nucleons suppress the energy tenfold. MICROSCOPE measured $\eta(\mathrm{Ti}, \mathrm{Pt}) = (-1.5 \pm 2.3 \pm 1.5) \times 10^{-15}$ [52], so the line read this way is out by more than a thousand of its errors. The same measurement holds any response of the weak share to the potential below about $2 \times 10^{-5}\,B$ at two errors ($2 \times 10^{-4}\,B$ with the suppression). A split that follows the local stay is therefore out unless its response is several hundred times weaker than the line's.

The three forms above read constants of the carrier instead, and free fall cannot reach them. The carrier's imbalance holds no stay, and it moves with light's dressing: both are set by

the spread of the bonds' sizes, and a shift of the carrier moves the imbalance about 1.5 times as fast as the dressing. The dressing in $K/16$ is the one light reads, and its constancy is measured: if it followed the Sun's potential, so would α, and clock comparisons bound that at $(c^2/\alpha)\, d\alpha/d\Phi = (14 \pm 11) \times 10^{-9}$ [53], which keeps the weak share's response under $10^{-9}\, B$. For the carrier's imbalance the same bound keeps it under $2 \times 10^{-8}\, B$. The occupation count's line stands if matter is read at its stay's value at balance rather than at its place. The ledger does not make that choice, and the printed share needs it. So free fall shows that the split's number is not a local stay; it does not choose among the three.

One sentence would cover the counts and the division together: *two columns held in two records are compared; two columns held in one record read their mean.* The first half is the pair reading of section 2.2, a body against the rest of the record, and gives the counts 8 and 4. The second half is the record that holds a debit with its credit on one account: in the mean of its two readings a turn's first order cancels except through the imbalance, which is the term above. This is a working statement, assembled with all three couplings in view. It is not derived.

Because the share hardly depends on the dressing, it asks for the hadronic input by itself: 0.027776 ± 0.000074, within 1.5 combined errors of the value light asks. The two errors share part of the carry, so this is a consistency, not two independent data: two of the package's equations take the same side of the disagreement between the hadronic determinations, and one of them reads no dressing. The share's own ask lies 0.50 errors below the 2026 lattice value and 1.4 above the data-driven 0.02760(10).

The quark masses. On the ledger's mass table [57], three times each quark's number at the cell, counted in golden steps above the electron, sits on a whole place plus a small term, and that factor of three depends on colour's stiffness at the cell. Run at the long-wave value, the six quark rows give a factor of 2.9970 ± 0.0047, 0.63 of its error below three. The dressing at which the factor is exactly three, found by running the table, is 1.040244 ± 0.000241 (figure 1); the long-wave value lies +0.63 from it, the carrier's own stiffness +2.1 and the mean +2.5. The six terms of that table were fitted to the quark masses, so this is a check on fitted terms, not on fitted light; it is the one other measured equation that asks for the value of K, and the computed K lies between the two asks.

Two numbers it was never compared with. The same package predicts two quantities that played no part in choosing anything in it. With the fine-structure constant, the Fermi constant and the top and Higgs masses as measured, and the hadronic input, the strong coupling and the Z mass solved so that light, colour and the weak share hold at the cell, it gives the W mass as 80.3512 ± 0.0048 GeV and the leptonic effective weak mixing angle as $\sin^2\theta_{\mathrm{eff}} = 0.23160 \pm 0.00005$. Both were computed, with the rule for scoring them written down, before they were compared. The 2026 averages are 80.3625 ± 0.0077 GeV, 1.25 combined errors above, and 0.23150 ± 0.00013, 0.68 below [16]; both pass. With the matching's second order, the computed part in the centre and the parts not computed in the error, the same route gives 80.3525 ± 0.0051 GeV and 0.23157 ± 0.00006, 1.09 and 0.48 combined errors from the averages. The Standard Model alone, on the same inputs, sits at 1.37 and 0.86 errors, so these two numbers test the package less sharply than the weak share does. They do test that the

determination	$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$	light	α^{-1}
lattice, Conigli et al. 2026 [38]	0.027821(49)	−1.71	137.020(9)
lattice (Mainz 2022), Erler et al. 2024 [42]	0.02794(9)	+0.01	137.036(14)
lattice (BMW), Erler and Ferro-Hernández 2023 [41]	0.027842(63)	−1.22	137.023(11)
lattice, Mainz 2022 [39]	0.02773(15)	−1.32	137.007(22)
lattice (BMW 2017), de Blas et al. 2022 [40]	0.02766(10)	−2.51	136.998(15)
world average, 2024 review [15]	0.02783(6)	−1.41	137.021(11)
e^+e^- and 2023 pions, Di Luzio et al. 2024 [43]	0.02775(12)	−1.46	137.010(18)
e^+e^- and 2023 pions, Narison 2025 [45]	0.027707(45)	−3.50	137.004(9)
e^+e^- and 2023 pions, Narison 2023 [44]	0.027663(45)	−4.17	136.998(9)
e^+e^- data to 2019 below 2 GeV, Erler et al. 2024 [42]	0.02762(5)	−4.58	136.992(10)
e^+e^- data to 2019, Davier et al., input of the 2026 fit [16]	0.02760(10)	−3.05	136.990(15)
e^+e^-, Erler and Ferro-Hernández 2017, in the review's table [16]	0.02761(10)	−2.96	136.991(15)
e^+e^-, de Trocóniz and Ynduráin, in the review's table [16]	0.02754(10)	−3.59	136.981(15)
lattice $\Pi(0)$ at $m_\pi = 185$ MeV with perturbation theory, Dominguez and Hernández 2023, five flavours [51]	0.027486(73)	−5.17	136.974(12)
electroweak fit, input left free [16]	0.02734(33)	−1.80	136.954(46)

Table 2. Determinations of the hadronic input, light's pull at the cell on each (in standard errors, the carry's and the dressing's together), and the fine-structure constant the closure gives with each.

dressing enters the stay: with the rest undressed, the angle would miss by 4.0 errors. Against the earlier W-mass combination that includes the CDF measurement [37], the package, like the Standard Model, misses by 3.4. The Z mass the package solves for, with the Fermi constant setting the scale and the matching's second order included, is 91.2006 GeV. The measured value is 91.1876 ± 0.0021 GeV, so the package lies 13.0 MeV above it. The carry's own error is 5.0 MeV and the part of the second order not yet computed, with the count's choices at its jumps and ties, adds 6.9 MeV, so that is 1.48 combined errors. This is the 1.5-error gap between the hadronic inputs the weak share and light ask for, seen in the Z mass. It is not an independent test.

6 Where it does not close

The closure's error is the carry's, and the largest part of the carry's error is the hadronic input $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$: how much the quark loops screen light between rest and the Z. It cannot be computed in perturbation theory at low energy. It is determined two ways, from the rate at which electrons and positrons annihilate into hadrons and from lattice computation, and the two disagree. The disagreement is the same one that divides the predictions of the muon's magnetic moment: the 2025 consensus evaluation of that moment set the electron–positron data aside for its hadronic part until the conflicts among the data sets are understood [17]. Table 2 lists the determinations and what the closure gives on each.

On the determinations that rest on a lattice computation at the physical pion mass the closure holds within 1.8 errors on four of the five, one of which lies on the value it asks;

the fifth, the one that takes its low-energy part from the lattice data of 2017, lies 2.5 below. These determinations differ as much in how they carry the lattice data to the Z as in the data themselves: the Mainz data of 2022 give 0.02773 carried with the Adler function from $Q^2 = 5\ \mathrm{GeV}^2$, and 0.02794 when only the light quarks are taken from the lattice, at $Q^2 = 4\ \mathrm{GeV}^2$, and the charm and bottom quarks and all the running above are taken from perturbation theory (an earlier calculation of the same kind gave 0.02795(11), +0.09). On the 2024 world average and on one electron–positron evaluation with the 2023 pion data the closure holds as well. The other electron–positron determinations with errors of 0.0001 or less lie between 3.0 and 4.6 errors below, and a determination that takes from a lattice computation only the light quarks' correlator at zero momentum, at a pion mass of 185 MeV, and everything else from perturbation theory lies 5.2 below. A fuller census of thirty-three determinations and fit values, from 2001 to 2026, shows the same pattern.

The 2026 lattice determination [38] adds to its lattice part the running from $Q^2 = 9\ \mathrm{GeV}^2$ to M_Z^2, computed from the Adler function in perturbation theory and printed as 0.019304(35) (table 7 of the second paper cited). We recomputed this piece with the code the determination names, AdlerPy 0.1.3 [54], as the integral it states, with the inputs it cites from the 2024 lattice averages [55]: $\alpha_s(M_Z) = 0.1183$, $m_c(3\ \mathrm{GeV}) = 0.991\ \mathrm{GeV}$ and $m_b(m_b) = 4.171\ \mathrm{GeV}$. The installation reproduces the code's own published example to its last printed digit. We varied, one at a time, each setting the determination does not state: four or five loops in the massless terms, the electromagnetic correction on or off, two choices of the charm and bottom threshold settings, and the gluon and quark condensates on or off. Every one of these settings reproduces the printed steps of the same table from 5 to 12 GeV^2 to within 0.9×10^{-6}, and gives the piece up to M_Z as 0.019371 to 0.019379, which is 0.000067 to 0.000075 above the printed value. (Pole masses for charm and bottom do not reproduce the printed steps and are set aside.) Ending the integral at $Q = 90.0\ \mathrm{GeV}$ instead of M_Z gives 0.019294 to 0.019302, close to the printed value but 2.5 to 10.4×10^{-6} below it, outside the table's own agreement, so that end is not the cause as it stands. The cause is not known. If the determination's total moved by 0.00007, it would be about 0.02789(5): on the value this closure filed in advance, 0.027882 ± 0.000019, and 0.7 combined errors below the value it asks with the second order as computed, 0.027939 ± 0.000048 (section 7). Table 2 and the rest of this paper use the value as printed. The scripts that give every number in this paragraph are in the code archive.

The weak share, which does not involve the dressing, says the same in the adopted convention: it lands at −0.50 on the lattice input and at +1.42 on the data-driven input of the 2026 fit. The two rows share the convention, the scale and the carry, so this is one convention's verdict, not two independent ones; in minimal subtraction the same form of the share would side the other way. The read does not rescue the closure on the electron–positron side: the reads that would land there are not light's (section 2.4). The closure's verdict is the lattice's against the data's.

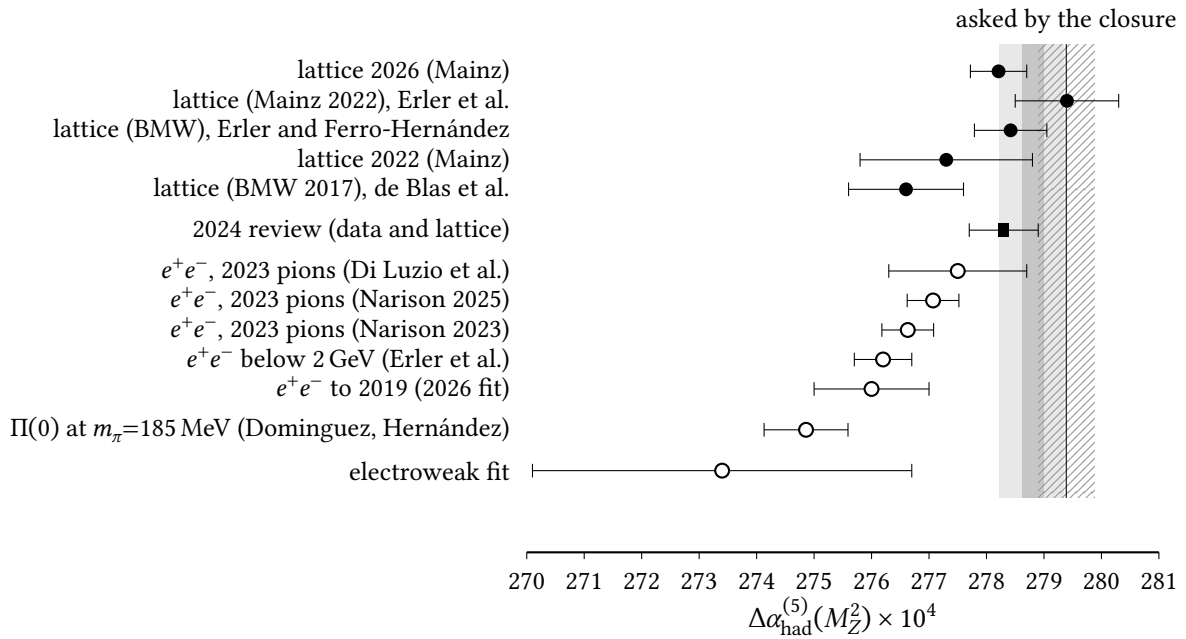


Figure 2. The hadronic input as determined (one standard error) against the value the closure asks: the hatched band is the value filed with version 1.8, 0.027939 ± 0.000048, and the line the value asked; the dark band is the value filed with version 1.1, 0.027882 ± 0.000019; the light band is the value filed with version 1.0, 0.027878 ± 0.000056. Filled circles: lattice computation at the physical pion mass; square: the 2024 world average; open circles: electron–positron data, a lattice value at a heavier pion mass with perturbation theory, and the electroweak fit. The filled circles lie within 2.6 combined errors of the value asked, one of them on it; the electron–positron values with errors of 0.0001 or less lie 3.0 to 4.6 below.

7 The test

The hadronic input, as filed with version 1.0. With α measured, the closure fixes the hadronic input it needs, the value that makes light land exactly. Version 1.0 filed it, on 14 November 2026, as

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027878 \pm 0.000056$$

(0.0278785 ± 0.0000561 before rounding), the error being the carry's other errors over the slope 10.888, with this rule: a new determination with an error of 0.00010 or less that lies within two combined errors of it passes; one that lies beyond three fails it, and with it the closure as stated, unless the carry itself is shown to be wrong; between two and three, it strains it. By that rule the closure fails on a new determination of error 0.00010 below 0.027534, or of error 0.00005 below 0.027653. (Computed from the printed 0.027878 ± 0.000056, the second boundary is 0.0276528, so a value between it and 0.027653 strains rather than fails.) That filing stands with its numbers.

The hadronic input on the corrected carry. The same rule on the corrected carry, with the dressing of section 2.3, is a second filing, also dated 14 November 2026:

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027882 \pm 0.000019,$$

with the measured strong coupling and top mass, or 0.027872 ± 0.000008 with the ledger's own. By it the closure fails on a new determination of error 0.00010 below 0.0275761, or of error 0.00005 below 0.0277209 (with the ledger's inputs, below 0.0275706 and 0.0277196). Both filings score only determinations first made public after 14 November 2026: those published before were in view when the rule was written, and they lie on both sides of the value asked (section 6). The test is of the package as stated, with the long-wave read. Changing the read after a determination would be a new hypothesis, not a rescue, and the read is not free (section 2.4). With the carry's three-loop completion taken from a shared start (section 3) the closure asks 0.027880 ± 0.000019 (0.027870 with the ledger's inputs); the difference is 0.07 of the filed error, and the filing stands as filed. With the matching's computed second order and the line for its uncomputed parts (section 2.5) the closure asks 0.027939 ± 0.000048 (0.027929 ± 0.000045 with the ledger's inputs). The filing 0.027882 ± 0.000019 stays as filed: it is scored by its own rule against every new determination, and the value the package asks at the time is printed beside it. With this version a new value is filed, dated and hashed, with its own rule below, and both filings are kept. No determination made public since 14 November 2026 has scored either of the earlier two.

The hadronic input with the second order computed. The same rule on this version's chain is a third filing, dated 15 November 2026:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027939 \pm 0.000048,$$

with the measured strong coupling and top mass and the line for the choices the record does not state in its error, or 0.027929 ± 0.000045 with the ledger's own. By it the closure fails on a new determination of error 0.00010 below 0.0276054 or above 0.0282722, or of error 0.00005 below 0.0277298 or above 0.0281478 (with the ledger's inputs, below 0.0275995 or above 0.0282583, and below 0.0277264 or above 0.0281314).

The strong coupling. The same package gives the strong coupling at the Z as 0.11836 ± 0.00003 in its convention at first order, thirty times sharper than its present measurement; with the matching's computed second order it is 0.11849 ± 0.00019 (section 2.5), a shift inside the rule's allowance. The rule for it was filed on the carry of version 1.0, where the package gives 0.11835: a new world average or determination $y \pm \sigma_y$ (minimal subtraction, five flavours, $\sigma_y \le 0.0005$) is scored by $t = |y - 0.11835| / \sqrt{\sigma_y^2 + 0.00003^2 + 0.0003^2}$, where 0.0003 is an allowance, fixed with the rule, for the carry's truncation and its matching at 200 GeV in this number: $t \le 2$ passes, $t > 3$ fails. For $\sigma_y = 0.0005$ the package fails below 0.1165984 or above 0.1201016.

The W mass and the weak angle. A later world average of the W mass with an error of 0.005 GeV fails the package as stated if it lies below 80.3311 or above 80.3739 GeV, and a later average of the effective angle with an error of 0.0001 if it lies below 0.231212 or above 0.231926. The Standard Model's own values on the same inputs sit close to the package's, so such a failure would first be the Standard Model's. Neither replaces the hadronic test.

A second measured equation: the muon. The quark loops that screen light between rest and the Z also make the largest hadronic contribution to the muon's anomalous magnetic moment, $a_\mu^{\mathrm{HVP,LO}}$. Both are integrals of the same function, the vacuum polarization of the colour-charged quarks, weighted differently: the muon reads 93 per cent of it below $Q^2 = 0.25\ \mathrm{GeV}^2$, the running of α reads it up to the Z. The closure's asked input therefore implies a value of $a_\mu^{\mathrm{HVP,LO}}$, and the measured muon tests it. The rule was fixed before computing: the comparison is with what the measured anomaly asks of $a_\mu^{\mathrm{HVP,LO}}$ once every other contribution of the 2025 Standard Model evaluation is subtracted, and a pull beyond two standard errors strains the closure, beyond three fails it.

The measured anomaly, $a_\mu = 116\,592\,071.5(14.5) \times 10^{-11}$ [46], less the other contributions of the 2025 evaluation [47], asks $a_\mu^{\mathrm{HVP,LO}} = (717.05 \pm 1.76) \times 10^{-10}$. To turn an asked input into a value for the muon we need the shape of the vacuum polarization below $Q_0^2 = 9\ \mathrm{GeV}^2$. Above it the running is perturbative (0.019304 as the lattice determination prints it, see section 6, plus 0.000045 from the space-like to the time-like Z), so the value filed with version 1.1, 0.027882 ± 0.000019, fixes the part below it: 0.008533, which is 0.72 per cent above the lattice value 0.008472. We take the shape from the same lattice group, which also gives $a_\mu^{\mathrm{HVP,LO}} = 724.5(7.1) \times 10^{-10}$ [49], and scale the muon's value by the same 0.72 per cent. The two lattice quantities are strongly correlated (a correlation of 0.8 has been published between a lattice muon value and its running at the Z [42]), so their ratio is sharper than the muon's value alone: its error is 0.71 per cent against 0.99. The filing then implies

$$a_\mu^{\mathrm{HVP,LO}} = (729.7 \pm 6.2) \times 10^{-10},$$

1.97 standard errors above what the measured muon asks: inside the rule, at its edge. With the correlation set to one the error is 5.5 and the pull 2.21, a strain. With the ledger's own strong coupling and quark masses in the carry the filed input is 0.027872 and the pull 1.77 (2.00 at full correlation). On the value filed with version 1.0 the pull is 1.57. Against the 2025 lattice average, $713.2(6.1) \times 10^{-10}$ [47], it is 1.90. On the value the closure asks with this version's second order, 0.027939 ± 0.000048, the muon's part is $(734.6 \pm 7.3) \times 10^{-10}$, 2.34 standard errors above what the measured muon asks (2.13 to 3.21 over the readings): a strain; with the correlation set to one, 2.54. With the perturbative piece above $9\ \mathrm{GeV}^2$ as we recompute it (section 6), 0.019371 to 0.019379 instead of the printed 0.019304, the pull is 1.49 to 1.58, and it passes.

The lattice's shape matters as much as the number. Read the same way, every asked input between 0.02759 and 0.02788 would pass at the error filed with version 1.1 (0.02757 to 0.02790 at this version's), so the muon does not by itself choose between the determinations of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$. What separates them is the shape: the 2020 evaluation of $a_\mu^{\mathrm{HVP,LO}}$ from electron–positron data, $693.1(4.0) \times 10^{-10}$ [48], lies 5.5 standard errors below what the measured muon asks, and the lattice evaluations lie within about one.

On the value filed with version 1.1 the closure asks 0.000061 more of $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ than the lattice determination of 2026, and the muon reads only the part of that excess that sits below $9\ \mathrm{GeV}^2$. With full correlation the muon passes if no more than 0.000047 of it sits there; at least 0.000014, nearly a quarter of it, must then sit in the perturbative running above $9\ \mathrm{GeV}^2$, where the muon does not look. If the whole excess sat there, the muon's pull would be +1.2 (+1.3 with full correlation): what remains is the lattice's own muon value, which lies above

the measured muon's ask. On the value asked with this version's second order the excess is 0.000118; the muon passes with full correlation if no more than 0.000074 of it sits below 9 GeV^2, so at least 0.000044, 37 per cent of it, must sit in the running above. Our recomputation of that running (section 6) gives 0.000067 to 0.000075 more than the printed value.

8 The chain, step by step

The table lists every step from the ledger to light's stiffness at the cell, and the carry from there to rest, with its status in one word. *Derived*: follows from the steps above it and the premises named, and is conditional on those premises where they are readings. *Reading*: an identification adopted and built on, not yet derived; what would derive it is named. *Checked*: what the step is, with every alternative counted and missed by measurement; the measurement was in view when it was first found. *Borrowed*: standard physics, used as it stands.

step	status	what it rests on, or what would derive it
the cell's edge $\lambda = 2\sqrt{\ln 2}\,\ell_P$	reading	one bit of record per λ^2 of area, fixed on gravity, not on light
the one bit a face counts gravity's whole stiffness, the fields' closed histories included	checked*	under the count of the matching the known fields' closed histories supply 1.57 per cent of that stiffness; if the bit counted the record's own stiffness only, the cell would be 0.78 per cent larger and light would stand at −2.20; counted and missed through the ledger's law for the electron's mass in units of gravity, which has no free number: it holds at 0.81 of G's errors with the whole stiffness and misses by 689 with the record's own stiffness only; *that law is a working one, fixed on one datum, and measured G alone reads neither
the lattice of events, 24 bonds	reading	the ledger's postings, four accounts with two columns; given it, the 12 bonds per event and the triangles are counted
the cost $J = \cosh \ell - 1$	derived	uniqueness of the reciprocal cost [20]
the cost acts as a weight (coefficient one)	checked	not a continuous setting; counted and missed: the cost in the amplitude, a coefficient of two (light 216 errors low), and a coefficient of one half (light 419 errors high); light locates it at 0.9960 ± 0.0023, light's shortfall (section 9); derived if the record's weights are shown to be its costs
each bond's fluctuation, $\langle \ell \sinh \ell \rangle = 1/12$	derived	the identity on the weight (section 2.3)
light reads the relaxed long-wave value, not the mean or the carrier's own stiffness	derived	what a coupling is: the coefficient in light's law at long waves, its links obeying their law in the carrier's weights; that energy splits into the relaxed cost and a part that does not depend on the loop value

step	status	what it rests on, or what would derive it
light reads the carrier without weighing it	reading	the same reading as "the links' own turns are not summed as weights at the cell"; summed with the carrier, light's turns would move its coefficient to about 35/24 and put colour ten errors low
the triangle weighs its links' mean	reading	each link reads a loop's whole turn (parallel); given it, the lattice's symmetry and the balance value force the mean; shared in series the harmonic mean, 17 errors low
$K = 1.040396 \pm 0.000002$	derived	computed; an earlier computation gives 1.040399 ± 0.000005
$1/g^2 = 2STK$ on the 24 bonds	derived	the lattice's triangles and their second moment
the ratio of light's count to colour's, two	derived	the ratio of their generator traces, given that light turns one of a body's two columns and colour the three partners; the rule gives light's and colour's totals only
the pair reading: an entry is one comparison of its two sides, priced once	checked	what an entry is: its debit read against its credit, $J(z^2)$, sets $S = 4$ and leaves K unchanged; priced as two events, the sum applied inside one entry, light is 5,547 errors low (the record's own sums add postings, which is the unit), as nothing it has no stiffness, read on one side or with three senses thousands of errors off; under the record's law of composition, a premise, one record of the two sides costs nothing or $J(z^2)$, never the sum; measured $S = 4.0006 \pm 0.0004$ (light) and 4.003 ± 0.005 (colour); the credit side is the whole rest because nothing less balances the entry; first found with the couplings in view; it also gives the matching's unit (with an entry's two sides priced apart the end would miss by 49 errors)
the imaginary part of the cost reads the columns' total turn only	derived	the cost of a complex ratio, $\cosh(u + i\varphi) = \cosh u \cos\varphi + i \sinh u \sin\varphi$: on the two columns the imaginary parts read the total turn and cancel on the weak turn; the term is $\tanh u$ of the real one and vanishes at balance
a record holds each link's unseen part	checked	counted and missed: a uniform hidden weight would stop unit charges from propagating, and free charges exist; the quarks' thirds need it in any form
no matching constant	derived	the summed count (section 2.5)

step	status	what it rests on, or what would derive it
the links' own turns are not summed as weights at the cell	reading	chosen with the couplings in view; a lawful weight sees no turn; summed, the turns would add constants to the matching that measurement excludes, and would make light weigh the carrier (colour ten errors low); summed under the pair alone, colour's turns on the 24 bonds would sit at the half turn (a triangle's mean reading −0.19, against +0.89 under four times the one-sided cost), so colour's carry from the cell rests on this reading too
a posting that leaves no record is in the least state of the equal-weight action, in entries and radians	checked	sets the matching's unit; the principle is in the ledger's posting law, and the number one half is on the record beforehand as the cost of one posting where a debit posting with a credit posting costs one; counted and missed by the end the three couplings ask: the record's own convention, strain one (5.0 errors), the other values it orders, one golden rung, one bit and one coin a posting (44 or more), an entry's two sides priced apart (49), the turn in whole cycles (119), the square of the least state (44), and every other value found on the record (more than eight); a cost of $\ln\phi$ stands 2.8 errors off; $71/144$ is counted and not missed (1.2 errors); one half is the ledger's own value; the radian, the bond and the least state were chosen with the end in view; measured: one entry 0.519 ± 0.013 against $\frac{1}{2}$ (the same datum as light's and colour's landings)
the inside of a history that leaves no record is not on the record's places	reading	lattice places inside such a history are refused by gravity's wave speed and the graviton's mass (no coupling enters); with it every walk on the bonds is excluded, the one form that fits the couplings better among them
the unrecorded count is summed	checked	the count of a history that leaves no record is not on the record, so it is summed, every whole number of units weighing one over itself; that gives the matching no constant at first order and one end for every spin; counted and missed: the lattice read as a regulator (light thousands of errors low), minimal subtraction at the cell (light more than seven errors low), and a cut on each line of a loop (21 errors from the one combination of the couplings that does not involve the end, which stands 1.8 from zero); identified with the couplings in view; to be derived from "nothing outside the record" on weighted histories

step	status	what it rests on, or what would derive it
gravity's strain adds nothing at the cell	checked	a strain is the record's own geometry, sourced and never summed; counted and missed: the strain's own histories summed like a field's, which in a cutoff scheme would raise light by a factor 1.41, where two errors leave room for a relative addition between −0.00003 and +0.00033; identified with the couplings in view; to be derived from the strain's own law
the carry from the cell to rest	borrowed	the Standard Model's running, three loops with computed shifts; the ledger has the two ends, not yet the passage
no other charged body below the cell	reading	the mass table's bodies; tested by every search for new charged or coloured states
the time bond is dressed like a posting bond, so a rest weighs K_m	derived	the lattice's symmetries
a move that closes no loop keeps weight one	derived	light's relabelling freedom, given that a move reads the carrier only through the light it closes (a reading that moves the share by 0.03 of its error)
a weight acts on odds	reading	adopted with the share in view; derived if the record's move law is shown to normalize over the sixteen moves
the stay $K_m/(K_m+3)$, $c = 12/16$	derived	from the rows above, and conditional on them
the weak share's count: each carrier's index, matter at its moving share, the scalar's weight by the line	reading	adopted with the measured share in view; the division of light is the readers' mean split number p (the rule's one half assumes single-column readers only); the value of p, 0.0650 measured, is not derived; three forms land it
the columns' total turn costs one number on every triangle, the mean squared imbalance $t = \langle \tanh^2 u \rangle = 0.0695885$, as odds: $p = t/(1+t)$	reading	located with the share in view and not adopted in place of the occupation count: share 0.4674695, −0.67; one of 88 values tried, 65 of them from the carrier; three forms land (−0.50, −0.39, −0.67) and the share does not choose; a cost bond by bond does not give it (11.1 errors low at long waves); derived if a record holding both columns is shown to weigh the carrier's mean squared imbalance against a single column's one, the weights totalling one
two columns held in two records are compared; two columns held in one record read their mean	reading	a working statement that would give the counts and the division of light together, assembled with all three couplings in view; its first half is the pair reading; derived if the ledger's cost is shown to compare records, with the credit side the whole rest of the record

A step that is a reading is called one wherever it appears in this paper. The readings were

identified with the couplings in view; section 4.1 counts what that costs. Read on one side, the same structure gives $2K$ for light and K for colour, which measurement excludes, and summing the links' own turns at the cell would give the matching constants that measurement excludes: the counts are what an entry is, checked by measurement, and the matching rests on steps that are checked or read, not on the structure alone.

9 What it means, and what is still owed

The fine-structure constant is light's turn stiffness, read at rest. At the ledger's smallest cell that stiffness is a whole count times one computed number, and the number comes from the ledger's own carrier: twelve bonds to an event, the cost of a comparison at coefficient one, and the stiffness a long wave reads from it. Carried down to rest, it is $1/137.020$. The magic number is a count and a stiffness.

Several steps of the chain follow from stated premises: the ratio of light's count to colour's, which stiffness light reads, the absence of a constant in the matching once the unrecorded count is summed, and the dressing of a rest. Each is conditional on its premises, and the premises that carry the physics are readings, several of them now checked, except the counts: an entry is one comparison of its two sides, priced once, and light and colour confirm it. What remains to derive is short and exact: that a posting which leaves no record is in the least state of the equal-weight action, with the radian, the bond and the least state as chosen, which sets the matching's unit; that the cell's one bit a face counts gravity's whole stiffness; that a record holds each link's unseen part; that the count of an unrecorded history is summed; that the links' own turns are not summed as weights at the cell; that gravity's strain is never summed; that the cost acts as a weight at coefficient one; that each link reads the whole of a loop's turn; and the readings that give the weak share's form, a weight acting on odds among them. The division of light between the weak link and hypercharge is the readers' mean split number, and its value is not yet derived. Light's landing can be read back as the number of senses (4.0006 ± 0.0004), one entry's cost (0.519 ± 0.013 against $\frac{1}{2}$), the weight's coefficient (0.9960 ± 0.0023) or the postings to a unit (7.70 ± 0.20); these are one datum re-expressed with the other readings held fixed, not four confirmations. That datum is light's shortfall, 1.71 of its errors: a tension, one number whether it is read as the dressing left free, the coefficient light locates or a common strain, and inside the carry's error. The next hadronic determinations will move it, and so may the matching's second order (section 2.5). Colour's own landing puts the senses at 4.003 ± 0.005. The carry from the cell down to rest is the Standard Model's computed running, borrowed whole: the ledger has the two ends, and the passage between them is still the borrowed calculation. Its error is the closure's error, and its hadronic part decides the test filed in section 7.

The pricing that sets light's count also gives the matching's unit its number. Costed on its pair, a comparison presents the square of its ratio. A hop's turn θ presents 2θ, which is the count's four. A posting's golden entry presents ϕ^2, which costs one half. A hop is one entry; a recognition's debit posting and credit posting are two, and they add to one. So two numbers of the closure, light's four and the unit's one half, rest on one pricing where they rested on two readings. The measurements are the ones already given. With the entry's two sides priced apart, the sum applied inside one entry, the count would be two and one entry would cost $2J(\phi) = 0.236$: the logarithms would end at $0.69\,\hbar c/\lambda$, 49 errors from the end the

three couplings ask. Its source is the entry's own structure: one comparison of its two sides (section 2.2).

Code and data

The computation of the carrier's three stiffnesses (the mean loop weight, its own stiffness, and light's long-wave stiffness at all orders and at second order), the stored runs, their analysis and the figure of their dependence on the lattice size are public at

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https://recognitionphysics.org/publications/fine-structure-constant/
                FineStructureFromTheLedger-code-v1.4.zip
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and in the paper's Zenodo record, doi:10.5281/zenodo.24227501, which resolves to its newest version. They are written in Python with NumPy; each run checks the lattice, and each run that solves for light its solver, against the definitions before it starts, and the archive lists the command for every run and the checksum of every file. Archive version 1.2 added the computation of the matching's second order (section 2.5), the recomputation of the 2026 lattice determination's perturbative piece and an independent rerun of it (section 6), the menus and chances of what the agreement weighs (section 4.1), the weak share's form against its ask with the bounds from free fall and from clocks, and the carry from the Z to the cell, with its code from the companion paper on the mass table, that gives the W mass, the weak angle and the Z mass (section 5). Archive version 1.3 added the run behind every number on light's and colour's side (the one this paper prints, with the runs for the range of readings beside it), the matching's second order where the read link meets the exchanged link's own line and on the scalar's closed histories, the second order where a field's quanta meet one another, colour's turns on the twenty-four bonds and the hadronic input's table and bounds, and brought the electroweak, free-fall, weak-split and odds computations to version 1.8's numbers. Archive version 1.4, which comes with this version of the paper, prints the muon's passing ranges rounded inward with their exact edges, and states which value of light's stiffness the runs of colour's turns used, with a check that no printed digit depends on it; it also holds the computations that draw the two figures.

About this version

Version 1.8.1 (16 November 2026) corrects version 1.8 of 15 November; no number of the result moves. Figure 1 drew light's two asks at version 1.1's values (on the lattice hadronic input 1.04048 ± 0.00007, on the data-driven input 1.04078 ± 0.00014) while its caption gave version 1.8's; it now draws version 1.8's, 1.04056 ± 0.00009 and 1.04086 ± 0.00015. Both figures are now drawn directly from the numbers the paper prints, with no coordinate set by hand.

The other corrected numbers now read as version 1.8's run gives them; each is given here with the value version 1.8 printed. Light's number of senses in the chain's table (section 8) is 4.0006 (printed 4.0005), as section 2.2 already said, and colour's is 4.003 there and in section 9 (printed 4.002). Three bounds were rounded the wrong way: the lattice determinations lie within 1.8 errors on four of the five (printed 1.7; one lies at 1.71), in the abstract and in section 6; figure 2's filled circles lie within 2.6 combined errors (printed 2.5; one lies at 2.51); and the muon's

passing range at this version's error is 0.02757 to 0.02790 (printed 0.02756 to 0.02791, rounded outward). Restatements of the asks that had kept a first-order or version 1.7 value now stand on this version's run. If the cell's bit counted the record's own stiffness only, light would stand at −2.20 (printed −1.77; sections 2 and 8). Light less twice colour, which contains no dressing, is measured at -0.0042 ± 0.0099 (printed -0.0034 ± 0.0098; sections 2.2 and 2.5); through it the coefficient is located at 0.964 ± 0.085 (printed 0.971 ± 0.084) and the end at 0.98 (printed 0.99) in units of $\hbar c/\lambda$. With the weak share in, the neighbouring end gives $\chi^2 = 16.1$ for three (printed 19.9) and still does not land. Within two errors the closure leaves room for a relative addition between −0.00003 and +0.00033 (printed −0.00007 and +0.00022; sections 2.5 and 8). Colour's window for K is ±0.0013 (printed ±0.0012; section 5). The harmonic mean misses light by 17 errors (printed 15; sections 2.4 and 8). For heavy neutral fields the sum of $y^2 \ln(E/M)$ stays below about 9 (printed 11; section 3). In section 2.4 the computed alternatives still carried version 1.7's misses of light: one link's weight misses by 5.7 to 7.4 errors (printed 4.7 to 6.2), the geometric mean by 9.6 (printed 8.1), the harmonic mean by 17 (printed 15) and the product by about nine hundred (printed eight hundred). In section 4.1 some figures were computed with the earlier value of the carrier's own stiffness, 1.040747, where the paper prints 1.040759, and now use the printed one. With the correlated chi-square as the measure the second of the three chances is 0.038 (printed 0.039). The ten-choice menu gives 0.0065 (printed 0.0066), in the table and in the text; the menu with the hadronic input also a choice gives 0.042 (printed 0.043) and the wider menu 0.076 (printed 0.077). On the errors without the line the second chance is 0.051 (printed 0.052). On the ten-choice menu the couplings put 0.94 of the weight on pure logarithms, 0.62 on the own stiffness and 0.28 on the long-wave value (printed 0.95, 0.68 and 0.24).

In section 7 the muon paragraph's first computation is now named for what it is: it uses the value filed with version 1.1, not this version's ask, which the paragraph gives next. The note on version 1.7 below again prints version 1.7's own count, 4,883 errors of two events, where version 1.8 had put its 5,547. Section 2.5 now prints the second order's constants in one place: for each of its five computed parts the constants under the count and under minimal subtraction, where its logarithm ends, light's slope and the shift of light's ask, with the sum that rebuilds the result from the printed numbers. The code paragraph named archive version 1.2; this version comes with archive version 1.4, and the paragraph now says what versions 1.3 and 1.4 added.

Version 1.8 (15 November 2026) replaced version 1.7 of the same day. Four of version 1.7's figures were wrong, and are corrected here first. Section 2.5 said that the two parts of the second order not computed, added plainly instead of in quadrature, would give 0.0120 in $1/\alpha$ and 0.0140 in all; those were the plain sum of all four parts then not computed, and for the two it named the plain sum is 0.000889 in light's ask, 0.0112 in $1/\alpha$ and 0.0133 in all (the quadrature figures, 0.0079 and 0.0107, stand). The chance of an accidental fit with the search, on the menus fixed before the count, was 0.021, not 0.022, wherever it was printed (the abstract, section 4.1 and the note on version 1.7). The sentence on the cycles of three postings, and the abstract's one form that fits slightly better, carried version 1.6's figures: on version 1.7's asks the cycles were at −0.84, −0.34 and +0.01, and four of the thirty-two forms fitted better, all of them walks on the bonds, so the exclusion and its conclusion stand. The Z line was computed with the dressing filed with the prediction, 1.040399, rather than the paper's 1.040396; on the paper's it reads 91.1968 GeV, 9.2 MeV above the measured value and 0.87 combined errors

(printed 91.1966, 9.0 and 0.85). The matching's second order is now computed where the record settles it: the histories in which the link that is read meets the exchanged link's own line, the scalar rings, and those in which the links' own quanta meet one another, whose count follows from two facts the record fixes and leaves two choices open, carried in the error. With them the result is $1/\alpha = 137.0199 \pm 0.0094$, and the measured value lies 1.7 errors above it (version 1.7: 137.0234 ± 0.0107); every number that rests on the asks is restated on them. One pole term of a published two-loop weight is recomputed (section 2.5). Thirteen more values were set against the weak share since version 1.7 (eighty-eight in all, fifteen within one error on this version's ask). The far-share form's condition is stated: it holds while a transfer weighs one. Colour's carry from the cell is said to rest on the reading that the links' own turns are not summed as weights there. On this version's ask the muon's leading hadronic part strains. The value of the hadronic input filed in advance stays and is scored as filed; a new value is filed with this version, and both are kept. The weights of the ten-choice menu now lean to the carrier's own stiffness over the long-wave read that the derivation takes.

Version 1.7 (15 November 2026) replaced version 1.6 of the same day. More steps of the chain are marked checked, each with its alternatives counted and missed by measurement: the matching's unit (one value, 71/144, is counted and not missed, and one half is the ledger's own value), the summed count of an unrecorded history, gravity's strain, the cost as a weight at coefficient one, and a record holding each link's unseen part; the cell's one bit a face is checked through the ledger's law for the electron's mass, which is fixed on one datum. The 4,883 errors of two events are the sum applied inside one entry; the record's own sums add postings, and that is the unit. The weak split's number is stated as three located forms, level, each needing one statement the record does not now make (for the carrier's form, one weight on every triangle), and free fall shows that it is not a local stay. A correction: the count of values within one error of the share, printed as eight since version 1.2, was eleven on the error printed then; on this version's error it is twelve of seventy-five, and three of the twelve have a reading behind them. Light's shortfall is named as a tension. The matching's second order is given its form: an exchanged link meets a closed history as the link that is read does, a working reading; the link's own stretch is counted as a line of its own quantum, the only complete count of the three; and where a field's quanta meet one another, the record fixes a condition the count must meet, and which count meets it is not yet settled. The 2026 lattice determination's perturbative piece was recomputed with the code it names (section 6). The matching's second order is derived in form where the record settles it and computed in part; with it the result moves to 137.0234 ± 0.0107, and the parts not computed are named and carried in the error. The weight of the agreement is recounted on the new errors: 0.21 given the coarse structure, 0.022 before the count, 0.00002 after the derivations.

Version 1.6 (15 November 2026) replaced version 1.5 of the same day. Version 1.5 called the counts readings identified with the couplings in view and left open whether an entry's two sides are priced once. That question was framed wrongly: an entry is one thing, one comparison of its two sides, and is priced once; asking whether its debit and its credit are one event or two adds a split the entry does not have. The counts follow from it, and the chain's table marks them checked. Every other pricing misses, two events by thousands of errors and nothing by all of light's stiffness; the measured couplings over K, 8.0006 and 4.0019, confirm it. It was first found with them in view. The matching now says that the whole-cycle form is the posting law's own, that the earlier attempt's one half is a field's cost and not the unit's,

and that the dressing left free and the coefficient light locates show one shortfall. No number of the result moves.

Version 1.5 (15 November 2026) replaced version 1.4 of the same day. Version 1.4 said that, of the values the record already held for a posting's cost, the nearest other was a cost of $\ln \phi$ and every other missed by twelve errors or more. A fuller search of the record found one value the couplings cannot tell from the half, $71/144$, and two at about eleven errors. No number of the result moves.

Version 1.4 (15 November 2026) replaced version 1.3 of the same day. Version 1.3 said that the record had left the strain a posting presents open between two values, that at the golden strain it priced a debit with its credit in one delta, and that it held nine numbers for a posting's cost. Each was wrong about the record. The record names more values, each fixed by one definition and none derived. Among them one posting costs one half where a debit posting with a credit posting costs one, which is the unit adopted here. Its price of one delta is for postings on one side of one account, not for a debit with its credit. The open step is stated sharper: if the cost of a record obeys the record's law of composition, an entry's two sides held in one record cost nothing or $J(z^2)$, never the sum, and the count of four rests on pricing one record, a reading the record itself leaves open. No number of the result moves.

Version 1.3 (14 November 2026) replaced version 1.2 of the same day. Version 1.2 said that the number of the matching's unit was newer than the comparison at the cell and that the one older number for a posting was $\cosh 1 - 1$; that was wrong, since the record held the unit's number before the end was located. Light's four and the unit's one half were stated to rest on one reading, the comparison of an entry's two sides, whose credit side is the whole rest of the record because nothing less balances the entry. The complex cost was written with its constant. No number of the result moved.

Version 1.2 (14 November 2026) replaced version 1.1 of the same day. The carry's three-loop completion is now taken from a shared start (+0.000005 in light's ask, not +0.000019), which moves the result to 137.0279 ± 0.0072 and light to -1.13; the filed input stands. The weight of the agreement now counts every form tried for where the logarithms end (0.014 before the count, 0.15 given the coarse structure), says that one of them, cycles of three postings, fits slightly better, and says why double entry excludes it. The pair reading is stated in the record's terms and closes colour beyond light; the matching's unit is set by the posting law, not by the pair; the division of light between the weak link and hypercharge is given its meaning, the readers' mean split number; the term of the cost that tells hypercharge's turn from the weak link's is identified, and a form built from the carrier alone lands the weak share at -0.49, one of three forms that land, with the count of what was tried; the matching's second order, the cell's one bit a face and the inside of a history that leaves no record are named; the end the three couplings ask together is given.

Version 1.1 (14 November 2026) replaced version 1.0 of the same day. The carry's error budget was recomputed with signs: one line of version 1.0's budget, the revision of the reference state, had been added in quadrature over two couplings whose moves cancel in light, so light's error is 0.000572, not 0.000811, and three computed shifts are applied. The dressing was computed a second time, independently, and the headline uses that computation. Together these moved the result from 137.0281 ± 0.0102 to 137.0277 ± 0.0072; on this budget the reported package lands light, colour and the weak share within two errors, not one. The census of the hadronic

input was completed: two lattice-based determinations lie above the value the closure asks, 0.02794(9) and 0.02795(11), and the sharpest electron–positron evaluations lie more than three errors below it. Colour's count is read on the pair as light's is, and the factor two between them is the trace of the generator. The second-order value 1.040378 is the one with a Gaussian covariance of the weights. New in version 1.1: which stiffness light reads, from what a coupling is, and the readings under the weighting and the matching; the weak share's count and the open problem it leaves; the count of the trials, with the chance conditioned on the coarse structure; two numbers filed before comparison and the solved Z mass; the muon equation; a second filing of the hadronic input; and the table of the chain's steps, each marked derived, reading or borrowed. The filing of version 1.0 stands with its numbers.

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