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The Fine-Structure Constant from the Recognition Ledger

Light's stiffness at the smallest cell, carried to rest

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Abstract

The fine-structure constant $\alpha \approx 1/137$ sets how strongly light and charge act on each other. It is measured to one part in ten billion; no accepted theory computes it. Here it is computed from the recognition ledger, with no continuous number fitted to light.

At the ledger's cell, $2\sqrt{\ln 2}$ Planck lengths on an edge, light's turn stiffness $1/e^2$ is a whole count, 8, times the stiffness a long wave reads from the ledger's own carrier, $K = 1.040396 \pm 0.000002$. It is computed on the lattice of events with no measurement in it, by two independent computations that agree. Carried to rest by the Standard Model's computed running, $8K$ gives

$$\alpha^{-1} = 137.0277 \pm 0.0072,$$

against the measured 137.035999177(21): sixty-one parts per million, 1.15 of the error. Colour lands with the same dressing (-0.40 standard errors) but admits every read of the carrier; the weak share (-0.31) tests its own form, not the dressing; the quarks' factor of three (-0.63) rests on fitted terms.

Some steps follow from stated premises: light's read from what a coupling is, and the absence of a matching constant if unrecorded histories are summed over every whole count. The premises that carry the physics, the counts among them, are readings identified with the couplings in view, and are named. The weight of the agreement is the coarse structure: light and colour at whole counts times one stiffness near 1.04 at a cell fixed by gravity, which a broad null gives once in 20 to 1,200. Given it, the precision adds little: a candidate would fit light as well with chance 0.85, light and colour 0.35, all three 0.04. On the menu fixed before the count the chance is 0.006, falling to 0.00001 as the derivations found afterward are granted.

The closure lands within 1.2 errors on four of the five lattice-based determinations of the hadronic input $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$, which carries most of the error, one of them above the value it asks, and 2.2 below the fifth. It is strained or fails on every electron–positron evaluation with an error of 0.0001 or less, by 2.7 to 4.9 errors. We file the value it asks, $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027882 \pm 0.000019$ (version 1.0 filed 0.027878 ± 0.000056); with it the muon's leading hadronic part sits 2.0 errors high, at the edge of its rule.

Version 1.1, 14 November 2026. What changed from version 1.0 is noted at the end.

1 The number

In 1916 Arnold Sommerfeld worked out how relativity splits the spectral lines of hydrogen into close pairs, the fine structure [1]. The size of the split was set by one pure number, which he called the fine-structure constant, $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$. It has no units. A civilization that counted in other units would find the same value.

Dirac's equation of 1928 explained the fine structure exactly [2], and it explained it in terms of α; it did not explain α. Arthur Eddington spent the rest of his life trying to: in 1929 he argued that $1/\alpha$ is exactly 136, and later that it is exactly 137 [3, 4]. Wolfgang Pauli worried at the number for decades; he died in 1958 in room 137 of the Red Cross hospital in Zurich [5]. In 1947 Lamb and Retherford measured a shift in hydrogen's levels [6], and Kusch and Foley an excess in the electron's magnetic moment [7], that Dirac's theory did not contain. Schwinger, Feynman and Tomonaga computed both from quantum electrodynamics, starting with Schwinger's $\alpha/2\pi$ [8]. Every one of those computations takes α as given. Feynman, who did as much as anyone to make them, wrote in 1985: "It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man" [9].

Measurement went on without explanation. The quantum Hall effect gave α from a resistance [10]. A single electron held in a trap gives it from the electron's magnetic moment, now to 1.1×10^{-10} [11]. Atoms recoiling from light give it from the ratio of Planck's constant to an atom's mass [12, 13]. The value recommended in 2022 is

$$\alpha^{-1} = 137.035999177(21) \quad [14].$$

At the energies of the large colliders the coupling was seen to depend on the length at which it is read: about $1/128$ at the mass of the Z boson [15]. That dependence is computed. The value at rest is not. In the Standard Model it is an input.

This paper computes it. Section 2 states what the recognition ledger gives for light at its smallest cell, which of its steps follow from stated premises, and which are readings. Section 3 carries the measured couplings from rest to that cell. Section 4 compares the two, states the fine-structure constant the ledger gives, and counts what the agreement weighs. Section 5 does the same for colour, for the weak share and for the quark masses, and gives two numbers the package was never compared with before they were filed. Section 6 says where it does not close. Section 7 files the numbers that test it. Section 8 lists every step of the chain with its status.

2 Light at the ledger's cell

2.1 The ledger, its bonds and its cell

The recognition ledger records every event as a posting on one of four accounts, each with a debit and a credit column [18]. Counted on both columns, the events form a lattice in four dimensions, the integer points together with the integer points shifted by $(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$. Its twenty-four shortest vectors, the bonds, are the sixteen half-integer vectors $(\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2})$, which are the eight debits and the eight credits, and the eight unit vectors $\pm e_i$, which are a debit with its credit on one account, its reverse, and the six edges of the record cube [18]. All twenty-four have one length, and the lattice's symmetries carry any bond to any other. Every event has twenty-four bonds and every bond joins two events, so there are twelve bonds for every event. Every bond lies in eight triangles of bonds, each event sits on thirty-two triangles, and every triangle closes one unit bond with two half bonds: neither kind of bond makes a triangle by itself. No arrangement of points in four dimensions has more than twenty-four nearest neighbours [21], so twelve bonds per event is a count of this lattice and not a setting; the smallest change a lattice allows, one bond more or fewer per event, would move light at the cell by more than forty of its errors.

Light is the loop law on the triangles of those bonds: a link carries a turn, and light's coupling is how stiff that turn is. Colour is the same loop law on an account's three partners.

The lattice has a size. One bit of record per λ^2 of area gives Newton's constant as $G\hbar = \lambda^2 c^3/(4\ln 2)$, so the cell's edge is

$$\lambda = 2\sqrt{\ln 2}\;\ell_P = 1.665109\,\ell_P \quad [19],$$

and the energy at which light is read at the cell is $E_P/1.665109 = 7.3322\times10^{18}$ GeV, with $E_P = 1.22089\times10^{19}$ GeV the Planck energy. The edge is fixed on gravity's bit and is used here unchanged. Where, at that edge, a coupling is read is the matching's question (section 2.5).

2.2 The counts

At the cell a link's turn stiffness, before anything dresses it, is a whole count, and the count is set by the lattice once two things are said: how a loop is read, and on which body. On the twenty-four bonds, for every uniform field strength the squared loop values of the triangles (the field strength's flux through each) sum, per unit four-volume, to exactly $F_{\mu\nu}F^{\mu\nu}$; the triangles' second moment is the same in every direction. So if a loop costs $S\cdot\frac{1}{2}\theta^2$ for a small turn θ in each direction in which the hopping body turns, each loop is dressed by K, and the body's generator squares to trace T, then the lattice's stiffness equals the continuum coupling for every field exactly when

$$\frac{1}{g^2}\Big|_{\text{cell}} = 2\,STK.$$

Colour is read on an account's three partners, where each generator has $T=\frac{1}{2}$. Light turns one of a body's two columns and leaves the other still, so its generator is a projector of rank one and $T=1$. The weak link and hypercharge, read on the same two columns, have $T=\frac{1}{2}$ each, and light's trace is their sum, $\operatorname{tr}Q^2 = \operatorname{tr}T_3^2 + \operatorname{tr}Y^2 = \frac{1}{2}+\frac{1}{2}$; that is the linear algebra of

light's trace, not a statement that each part has $4K$ (section 5). The ratio of light's count to colour's is therefore the ratio of their generator traces, two, for every S: a unit charge against the fundamental's one half, as it is between the $U(1)$ and $SU(N)$ loop actions of field theory at equal cost per turn.

What sets S is how a comparison is costed. The reading adopted here costs it on its pair, the debit reading over the credit reading. For a comparison z whose credit reading is z^{-1} the cost is $J(z/z^{-1}) = J(z^2)$, one formula for every comparison. For a loop the credit reading is the loop's reverse, so the pair reads the turn twice,

$$1 - \cos 2\theta = 4\,(1 - \cos\theta) - 2\,(1 - \cos\theta)^2,$$

and $S = 4$:

$$\frac{1}{g_3^2}\Big|_{\text{cell}} = 4K, \qquad \frac{1}{e^2}\Big|_{\text{cell}} = 8K,$$

where K is the one dressing both read (section 2.3) and $1/e^2 = 1/(4\pi\alpha)$ in the usual notation. Read on one side only the counts would be 2 and 1, and with three senses 18 and 9: light would then miss by thousands of its errors and colour by hundreds. The pair reading was identified with the measured couplings in view (the measured couplings over K are 8.0006 and 4.0019) and is adopted. It is one reading, shared by light and colour and, as section 2.5 shows, by the matching's unit as well: fixed on light, it closes colour and the end of the logarithms. On a bond of the carrier the same formula reads the bond's ratio squared, in the carrier's weight and in light's read alike; that only relabels the carrier's variable by two, and it leaves K unchanged. On one golden entry it gives exactly one half (section 2.5). What would derive it is not settled. A cost that depends on two kept records only through their comparison is free of a common change of reference, but that alone does not pick the pair: $J(\sqrt{d/c})$ is just as free of it, and on a recognition, whose credit is the debit's reverse, it is the one-sided reading. What picks the pair is a calibration: that two records compared, one of them at balance, cost the comparison cost J of the other. That calibration is the open step. Measurement makes that choice three times, on light, on colour and on the end of the logarithms; reading one side, each side against balance, or the product misses by thousands of errors.

The costs that give these counts do not see every part of a link. Light's pair does not see a half turn of light's link: $1 - \cos 2\theta$ vanishes at $\theta = \pi$. Colour's cost does not see the overall turn of the partners' link at all, so turning that link by any sixth root of one costs nothing: light's link, its determinant, turns by at most a half turn. If the ledger summed over these unseen turns, a history would survive only if, on every link, the unseen turns picked up by all the bodies crossing it cancelled, and neither a free electron nor a free quark would exist. So a record holds each link's unseen part. That is a second reading; the quarks' thirds need it whatever form the count takes.

Carried to the cell (section 3), the measured couplings give

$$\frac{1}{e^2} - \frac{2}{g_3^2} = -0.0033 \pm 0.0100,$$

which is colour's landing said again, not an independent check; what it adds is that the factor two, the trace, holds at the cell fixed on gravity with no dressing in it.

2.3 The dressing

The carrier is one real number on every event, the logarithm θ of its ratio. A bond b between two events carries the difference $\ell_b = \theta_x - \theta_y$, and the cost of a bond is the ledger's cost of a comparison, $J(e^{\ell}) = \cosh \ell - 1$ [20]. The carrier sits at weight

$$P[\theta] \propto \exp\Big(- \sum_b \big(\cosh \ell_b - 1 \big) \Big)$$

with coefficient one (section 2.4). Because the weight depends only on differences across bonds, the identity $\sum_x \langle \theta_x \, \partial S / \partial \theta_x \rangle = N - 1$ for a lattice of N events becomes, bond by bond,

$$\langle \ell_b \sinh \ell_b \rangle = \frac{N-1}{12N} \;\longrightarrow\; \frac{1}{12},$$

the twelve being the number of bonds per event; the bonds are alike, so the identity holds for each. That fixes how much each bond fluctuates. If the fluctuation of a bond were exactly Gaussian with variance s, the identity would read $s e^{s/2} = 1/12$, so $s = 0.0800633$, and the mean loop weight would be $\langle \cosh \ell \rangle = e^{s/2} = 1.040844$. Computed directly on the lattice it is

$$K_m = \langle \cosh \ell_b \rangle = 1.0408495 \pm 0.0000001,$$

a little above the Gaussian value because the bonds have thinner tails than a Gaussian: their fourth moment is 0.7 per cent below $3s^2$. That is the dressing a single loop at one place reads: the mean.

Light is a different field. It lives on the links, and its energy is $\sum_T w_T F_T^2$ over the triangles T of the bonds, where F_T is the field's value around the triangle and w_T is the mean of $\cosh \ell$ over the triangle's three links. Light's stiffness at long wavelengths is the energy per cell of a uniform field strength f, minimized over periodic link fields A:

$$K = \min_A \frac{1}{N} \sum_T w_T \big(f_T + (\operatorname{curl} A)_T \big)^2 \Big/ \frac{1}{N} \sum_T f_T^2 .$$

With every w_T equal to the mean, K would be the mean. Because the weights spread, the field relaxes into the softer triangles and K falls below the mean. On every configuration of the carrier the relaxation is bounded: the minimum lies between the harmonic and the arithmetic mean of the triangle weights. The arithmetic mean is the energy of the field at rest, which is one of the fields tried; the harmonic mean follows from the Cauchy–Schwarz inequality, because a uniform field strength has no divergence on this lattice (the eight triangles on a link carry fluxes that cancel). To second order in the spread,

$$K = \bar{w} - \frac{1}{N \bar{w} \sum_T f_T^2} \sum_q \operatorname{tr} \big(\Pi(q) \, \langle g \, g^{\dagger} \rangle(q) \big), \qquad g_T = (w_T - \bar{w}) \, f_T ,$$

with Π the projector onto the loop values a periodic field can make. With the covariance of the weights taken from a Gaussian carrier of the same bond variance ($s e^{s/2} = 1/12$), the gap on lattices of 4^4, 6^4 and 8^4 cells is 0.000466, 0.000472 and 0.000473, which gives $K = 1.040378$. With the carrier's exact covariance, the same expression evaluated on sampled configurations, the gap is 0.000466 and $K = 1.040384$.

At all orders the minimization is solved exactly. The carrier is sampled from $P[\theta]$ by hybrid Monte Carlo on periodic lattices of L^4 cells, $L = 4$ to 16. On each sampled configuration, for each of the six coordinate planes of the field strength, the periodic link field of least energy is found to a relative residual of 10^{-11}, and the results are carried to infinite volume in $1/L^4$. The gap is 0.000453, and

$$K = 1.040396 \pm 0.000002.$$

An earlier computation gave 1.040399 ± 0.000005. The two were written independently and agree within their errors; the code, the runs and the analysis of the computation used here are public (see Code and data, at the end). On the samples the identity $\langle \ell \sinh \ell \rangle = (N-1)/(12N)$ holds at every size, and each of the 181,200 minimizations lies inside its two bounds. Of the difference between the Gaussian second order and all orders, 0.0000197, the exact covariance accounts for 0.0000070 and the orders beyond the second for 0.0000127.

2.4 Which stiffness light reads

The carrier has three stiffnesses: the mean loop weight $K_m = 1.0408495$, the long-wave value $K = 1.040396$, and its own stiffness against a slow twist of its own field, 1.040759 ± 0.000002 (an earlier computation gave 1.040747 ± 0.000007). Which one is light's follows from what a coupling is. How a triangle weighs its three links, and whether light changes the carrier it reads, rest on two readings, stated as such below.

The read. A coupling is the coefficient of the squared field strength in a field's law at long waves. For light on the carrier that is the energy per cell of light's field at a uniform loop value f, with light's links obeying their own law in the carrier's weights. On one configuration of the carrier this energy splits exactly into two parts: the cost of f once the links have settled into the weights, and the cost of the field's departure from that settled shape, which does not depend on f. So the coefficient is the relaxed stiffness, the long-wave value, configuration by configuration. The mean loop weight is the cost of a uniform loop value whose links have not settled. It bounds K from above, and it is what one loop at one place reads, not what light's long wave reads.

Not the carrier's own stiffness. For a bond of size u carrying a turn φ, the comparison reads the complex ratio $e^{u+i\varphi}$, and $\mathrm{Re}\cosh(u+i\varphi) = \cosh u \cos\varphi = \cosh u - \cosh u\,(1-\cos\varphi)$. The turn enters as $\cosh u\,(1-\cos\varphi)$: with stiffness $\cosh u$, even in the turn, and never multiplying $\sinh u$, so no current runs between light's turn and the carrier's size. The carrier's own stiffness is its response to a twist of a field that lives on events, and such a twist has loop value zero around every closed triangle. It is never light's.

The weighting, a reading. We read each link as reading the whole of a loop's turn with its own stiffness, so that a loop's cost is the sum of what its links read: the links act in parallel. Given that, a triangle's weight that is unchanged when the lattice's symmetry cycles its three links, and equal to k when every link has stiffness k, is the arithmetic mean of the three. The alternative reading shares the loop's turn among the links in series, each taking a part inversely to its stiffness; it gives the harmonic mean, and light then misses by 22 errors. One link's weight alone breaks the lattice's symmetry, the product of the three is not k when every link has stiffness k, and the geometric mean is not a sum of what the links read.

Light reads the carrier without weighing it, a reading. Light's law depends on the carrier

through the triangle weights. If light's turns were summed as weights together with the carrier, the sum over light would leave on each configuration of the carrier a factor that depends on that configuration, and the carrier's law would shift: at first order its coefficient would become about $35/24$, K would be 1.027995, and colour, whose ask does not depend on the hadronic input, would miss by ten of its errors. We read the cell the other way: the links' own turns are not summed as weights there, the same reading that keeps light's own loops out of the matching (section 8). Nothing then multiplies the carrier's weight, and the coupling is the relaxed stiffness averaged over the carrier's own law. What motivates the reading is that light's turn term, a stiffness times $1-\cos F$, is not a cost that obeys the ledger's law, while a real cost that does obey it gives every turn the weight of the size alone [18]. That motivates the reading; it does not prove it.

Computed alternatives miss light on the carry of section 3: one link's weight by 6.4 to 8.6 errors, the geometric mean by 11, the harmonic mean by 22, the product by more than a thousand. The read and both readings were identified with the landing known. What the read adds is that it follows from what a coupling is; what each reading adds is a reason that does not mention a coupling, and an alternative that misses.

The coefficient. One identification is left under the read: each comparison weighs its history by e^{-J}, with J the comparison's own cost, so the cost acts as a weight at coefficient one. The cost's unit is not a setting: changing it relabels the carrier's variable, and the dressing does not see a relabelling. The one discrete alternative, the cost placed in the amplitude instead of the weight, would put light nearly three hundred errors low and colour sixteen. Light locates the coefficient at 0.9979 ± 0.0018; that is light's landing said for the coefficient, not a second confirmation of it. The matching's unit rests on the same identification, and through it the combination of light and colour in which the dressing cancels, whose error is mostly colour's, locates the coefficient at 0.971 ± 0.084.

2.5 Matching the lattice to the continuum

A stiffness on a lattice and a coupling in the continuum are compared through a matching, which in general has a scale and a constant. If the ledger's lattice were a regulator, with the fields' own fluctuations summed along its bonds, the constant would be large: the links' own loops summed at the cell take 2.54 from light and 1.52 from colour, and matter summed along the bonds adds 0.12 to light. Each moves light by hundreds to thousands of its errors, and the combination of light and colour that contains no dressing, measured at -0.0033 ± 0.0100, would stand at $+0.50$ with the links' loops summed. Read as a regulator, the lattice is excluded by measurement.

The reading adopted here counts the histories that leave no record differently. Such a history has no count on the record, and its count is read as summed: every whole number of units from one, each weighing one over itself. If $q(t)=\sum_i w_i e^{-T_i t}$ is the return weight of one unit, at $t=k^2+m^2$ for a wave number k and a rest m, the sum is $\sum_{n\ge 1} q^n/n = -\ln(1-q)$, and for small t

$$0 \;\le\; -\ln\big(1-q(t)\big) - \ln\frac{1}{T_1 t} \;\le\; \frac{2M_2 t}{T_1}, \qquad T_1=\sum_i w_i T_i, \quad M_2=\sum_i w_i T_i^2.$$

So a field adds its coefficient times one logarithm and nothing else, whatever the shape of one

unit, and the same for every spin. The comparison has no constant, and a constant common to every field is the same thing as a different unit: the constant and the scale are one number, the proper time T_1 of one unit, and the logarithms end at $1/\sqrt{T_1}$. In the language of the field-theory literature this is the dimensional-reduction scheme with every threshold a pure logarithm; it differs from minimal subtraction by the constant $C_A/(48\pi^2)$ in each $1/g^2$, where C_A is 2 for the weak group and 3 for colour [22].

Under the pair reading one unit is one recognition: each of the eight columns, four accounts with a debit and a credit, posted once. The four debits are an orthonormal frame of bonds, and so are the four credits, so one recognition's second moment is exactly twice the round one and its proper time is exactly λ^2: the logarithms end at $\hbar c/\lambda$. Each posting weighs one entry at one half, and that is the pair reading of section 2.2 again: one golden entry read on its pair, its debit side up one golden step against its credit side down one, costs exactly $J(\varphi^2) = \frac{1}{2}$. Read back from the measured couplings, one entry costs 0.510 ± 0.011; the entry read on one side only (0.118), or four times that cost at small size (0.472), is more than three errors off.

The measured couplings put the end at $(0.99 \pm 0.04)\,\hbar c/\lambda$ from light less twice colour, which contains no dressing, and at $(1.010 \pm 0.011)\,\hbar c/\lambda$ from light and colour with the dressing: 7.84 ± 0.16 postings of bond size to a unit, where the ledger has eight, and a unit proper time of 0.980 ± 0.020 in units of λ^2. Seven and nine are more than five errors away. One neighbour that is not a whole number of postings, the exact cost read as an amplitude (an end at $1.0305\,\hbar c/\lambda$), lands too, so the couplings fix the unit to two per cent, not to the integer by themselves. That neighbour, and the one that weighs a posting by the exact cost of its length as a weight (an end at $1.290\,\hbar c/\lambda$), are excluded without a measurement: a step weighed by the exact cost of its length obeys no law of comparison in a plane, whatever product term it carries, so only the plain law weighs where a posting goes. The first also puts the cost in the amplitude, which the carrier excludes (section 2.4). Minimal subtraction at the cell misses light by more than eight errors, and reading the coupling at the zone's edge π/λ by more than ninety. A cut on each line of a loop, which is what a regulator of fields does, would give scalars, spinors and vectors different ends; the measured couplings ask for one end for every spin, within 1.7 of the error, and the cut lies 41 errors away. That 1.7 is the largest pull among the couplings: it is light's shortfall, seen in the one combination of the three couplings that does not involve the end.

Gravity at the cell belongs to this matching. In units of the cell's area Newton's constant is $1/(4\ln 2) = 0.36$, which is not small. If the strain's own histories were summed like a field's, light would move by tens of per cent (by a factor 1.41 in a cutoff scheme [23]). Within two errors the closure leaves room for a relative addition between -0.00007 and $+0.00022$. The result therefore rests on the fourth of those statements: the strain is the record's own geometry, sourced and never summed.

Four statements carry the matching and are not yet derived from the ledger's foundation: that the count of an unrecorded history is summed, that a comparison is costed on its pair (which sets the unit), that the links' own turns are not summed as weights at the cell (section 2.4), and that gravity's strain is never summed. With them the matching has no free number.

3 The carry from rest to the cell

What the measured couplings are at the cell is a Standard Model calculation, and we use it as it stands. The state of the Standard Model at 200 GeV is taken from the interpolation formulas of Alam and Martin [28], which match the on-shell measured inputs (the fine-structure constant, the Fermi constant, the masses of the Z, the top quark and the Higgs boson, the strong coupling, the quark and lepton masses, and $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$) to the Lagrangian couplings at three loops and beyond. From there the three-loop renormalization-group equations [31, 32, 33] run every coupling to the cell. Three effects beyond that running are known and are applied to light's ask: the complete three-loop running of the Yukawa couplings and the Higgs self-coupling, as the reference code's own published running shows (+0.000019; [29]); the four-loop gauge terms (+0.00005; [34]); and the top quark's decoupling from light at order $\alpha\alpha_s^2$, which the two-loop matching at the Z [30] leaves out (−0.00006; [36]). Converted to pure logarithms, with the 2026 electroweak inputs, $\alpha_s(M_Z) = 0.1180(9)$ and the 2026 lattice value of the hadronic input, the measured couplings ask at the cell:

	asked at the cell	error
light, $1/e^2$	8.323827	0.000572
colour, $1/g_3^2$	4.163585	0.004975
weak share, g^{-2}/e^{-2}	0.4674982	0.0000583

Light's error has these parts: the hadronic input (0.000534), the strong coupling (0.000195), the top mass (0.000058), and, together 0.00004, the matching of the reference state (its last revision moved light by 0.000015, its interpolation formulas by at most 0.000017), what remains of the matching at the Z after its three-loop top term (0.00003), and the running beyond four loops (0.00001). Kept as version 1.0 kept it, with the revision of the reference state added in quadrature over the two couplings (0.000573), light's error would be 0.000810 and light would sit at −0.81 instead of −1.15, with $\alpha^{-1} = 137.0277 \pm 0.0102$ and the filed input 0.027882 ± 0.000056; the value does not move and the conclusion does not depend on the choice. We use the signed step because the revision changed the electroweak scale, which moves g and g' in opposite directions (by −7 and +2 parts in 10^5) and leaves light, which is fixed by α and the hadronic input, almost unchanged (+0.000015), as a change of the Z mass does (0.000004); the matching of light itself, which both versions share, is carried separately (0.00003). The weak share's error keeps that revision whole, because a change of the electroweak scale moves the split even when it leaves the sum alone. Colour's error is almost entirely the strong coupling's. The series is short: from one loop to two the running moves light by −0.0531, from two to three by −0.0015, and the four-loop terms by +0.00005, opposite in sign and a thirtieth of the three-loop term [35]. A series whose next term is computed, and is a thirtieth of the last, does not have the last term as its error.

The running contains no gravitational term. In a mass-independent scheme such as this one, Newton's constant can enter the running of a dimensionless coupling only multiplied by a squared mass of the theory ($Gm_t^2 \approx 2\times10^{-34}$) or by the cosmological term [25]; over the thirty-nine e-folds from the Z to the cell that moves light by less than 10^{-30}. With a momentum cutoff in a particular gauge, gravity's power-law correction would make light's stiffness 1.41 times larger at the cell [23]; it depends on the gauge [24], it is absent in dimensional regularization

[25], it does not act as a universal running coupling [26], and in an asymptotically safe completion it would add about 0.18 [27]. Those effects are the size of the higher-dimension terms at the cell. They belong to the matching (section 2.5), which compares the coefficients of the dimension-four term, not to the carry.

The carry assumes that no field carrying electric, weak or colour charge exists between the Z and the cell other than the Standard Model's. That is a premise, and with the closure it becomes a prediction. Every such field softens light at the cell with the same sign. One charged fermion at 1 TeV would move light by about a thousand of its errors, and one anywhere below 0.967 of the cell's energy (7.09×10^{18} GeV) would move it by more than one error. Heavy neutral fields count too when they bind strongly to the lepton doublets: within one error, the sum over them of $y^2 \ln(E/M)$ stays below about 7.

The value at rest follows from the value at the cell through one slope. The fine-structure constant enters the carry only through the coupling at the Z, $1/\alpha(M_Z) = (1 - \Delta\alpha)/\alpha$. Every vacuum term in $\Delta\alpha$ is itself proportional to α, so $\Delta\alpha/\alpha$ does not move with α, and a change in $1/\alpha$ moves $1/\alpha(M_Z)$ one for one. At fixed α the hadronic input moves $1/\alpha(M_Z)$ by $-1/\alpha$ per unit and light at the cell by -10.888 per unit. So

$$\frac{d(1/e^2)_{\text{cell}}}{d(1/\alpha)} = \frac{10.888}{137.036} = 0.0794553,$$

close to the one-loop value $1/(4\pi) = 0.0795775$.

4 The fine-structure constant

With $K = 1.040396$, light at the cell is $8K = 8.323168$, against 8.323827 ± 0.000572 asked. With the dressing's own error added, that is -1.15 of the error. Through the slope,

$$\boxed{\alpha^{-1} = 137.0277 \pm 0.0072}$$

against the measured 137.035999177(21). No continuous number in it was fitted to light: K is computed, the count is whole, the cell is fixed on gravity, and the carry is the Standard Model's own. With the earlier computation of the dressing, $K = 1.040399$, it is 137.0280 (-1.11).

When every input of the carry that the ledger has its own number for is taken from the ledger instead of from measurement (the strong coupling solved so that colour at the cell is exactly $4K$, which gives $\alpha_s(M_Z) = 0.11836$, and the quark masses of the ledger's mass table, with the top quark at 172.46 ± 0.37 GeV), light asks 8.323720 ± 0.000541 at the cell and

$$\alpha^{-1} = 137.0291 \pm 0.0068,$$

-1.02 of the error. The ledger's values move the result toward the measurement.

read of the carrier	K	earlier	light	α^{-1}	light's read?
long wave, all orders	1.040396	1.040399	-1.15	137.0277	yes
second order, own covariance	1.040384		-1.32	137.0265	approximation
second order, Gaussian	1.040377	1.040378	-1.42	137.0258	approximation
the carrier's own stiffness	1.040759	1.040747	$+3.92$	137.0643	no (§2.4)
mean loop weight	1.0408495	1.040851	$+5.19$	137.0734	no (§2.4)

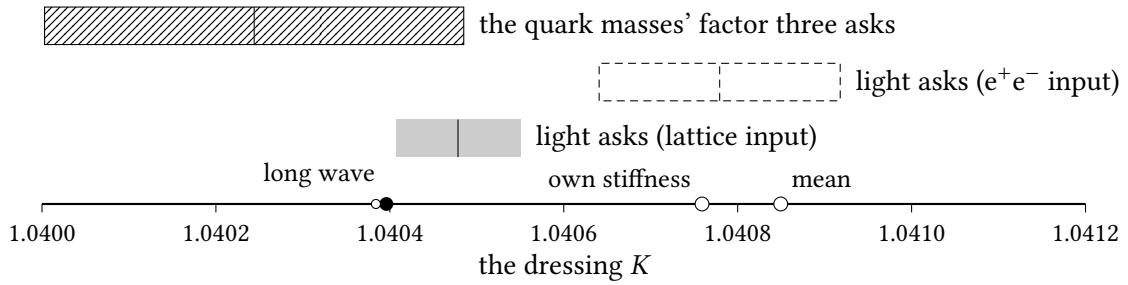


Figure 1. The dressing each measurement asks for, against the carrier's computed reads. Filled dot: the long-wave value, light's read; small circle: its second-order approximation; open circles: the carrier's own stiffness and its mean, which are not light's (section 2.4). Light's ask on the lattice hadronic input (grey) lies 1.15 of its error from the long-wave value. The quark masses' factor of three (hatched) asks a dressing within one error of it. On the data-driven hadronic input (dashed) light's ask moves up by 0.0003: there the own stiffness and the mean would land and the long-wave value would not, but neither is a read light can take.

4.1 What the agreement weighs

The result rests on discrete choices, and they can be counted. Ten were written down, each with its natural alternatives and equal weights, before any combination beyond the eighteen on record had been scored; the reported combination's agreement was already known when they were chosen: light's count (8, 2, 4, 16); colour's (4, 2, 8, 16); which stiffness of the carrier light reads (three); how a triangle weighs its links (five); the weight's coefficient (1, $\frac{1}{2}$, 2); the bonds per event (12, 4, 8); the matching constant (none, minimal subtraction, and three lattice-type constants); the reading scale (six multiples of $\hbar c/\lambda$); the form of the weak share (seven); and the dressing the share reads (three). That is 1,360,800 combinations, each scored on the carry of section 3. None lands light, colour and the weak share within one error. The reported combination lands all three within two (-1.15, -0.40, -0.31), and no combination fits better.

What the agreement weighs is how often a menu this large would produce a fit this good by accident. Let the three couplings lie anywhere within one per cent of their values, uniformly and independently, and ask for the chance that some combination fits them at least as well as the reported one does. Table 1 gives it for the menu above, for larger menus, and for the menus left once the derivations of section 2 remove alternatives.

Three numbers say what the agreement weighs. On the menus fixed before the count, the chance of an accidental fit this good is 0.0062 (every alternative the derivations were tested against; 0.0018 on the ten-choice menu). That null is generous to the result: one per cent is much wider than the spread of what the candidates predict. Conditioned instead on the coarse structure, with each coupling anywhere among the values the candidates predict for it (the thirteen computed stiffnesses of the carrier and the retired value 1.04108125, both conventions, the seven forms of the share), some candidate fits light as well as the reported one with chance 0.85, light and colour together with chance 0.35, and all three with chance 0.04 (0.12 if the share's range is taken from the three forms on record alone). After the derivations that were found once the agreement was known, the first chance falls from 0.00026 to 0.00001 as each is granted; the last value is the package's own chance, with nothing left to choose. Each derivation rests on premises that do not mention a coupling (sections 2.2 to 2.5), but each was

menu	combinations	chance
fixed before the count, with the reported agreement already known		
the ten choices above	1,360,800	0.0018
the alternatives each derivation of section 2 was tested against: the counts, the ends of the logarithms, the reads and weightings of the carrier	385,560	0.0062
the same, with the hadronic input also a choice	771,120	0.017
wider: counts up to 24, nine scales, forms of the share off the photon line, the hadronic input a choice	38,381,850	0.029
after the exclusions derived once the agreement was known		
the read, the matching constant, the ratio of the counts, the bonds per event, the time bond's dressing (and with it the dressing the share reads) and the ends of the logarithms that break a theorem derived; the weighting granted as a reading	324	0.00026
also: a weight acts on odds	270	0.00021
also: the dressing sits on loops, so the share's form is fixed	54	0.00005
also: the cost of a comparison is read on its pair, which sets the number of senses and the end of the logarithms together	2	0.00001

Table 1. The chance that some combination of a menu fits light, colour and the weak share at least as well as the reported package, if the three couplings had been anywhere within one per cent of their values. The upper rows were fixed before the count; the lower rows remove alternatives by derivations found after the agreement was known.

found with the agreement in view, so we give all three numbers and do not choose between them; a derivation made without the couplings in view is the test of that. The widest menu, which lets the read and the hadronic input be chosen after the fact, gives 0.029.

The precision of the agreement carries little beyond the coarse structure and colour. Given that light's coupling at the cell lies among the carrier's stiffnesses times eight at all, some candidate fitting it as well as the reported one does is what chance gives about five times in six. What carries the weight is that light and colour sit at whole counts times one stiffness near 1.04 at the cell fixed by gravity, together.

On every menu the couplings themselves select the counts 8 and 4, the coefficient one, the twelve bonds per event and the reading at $\hbar c/\lambda$, and they leave no weight on any lattice-type matching constant. On the ten-choice menu they lean to pure logarithms (0.81 of the weight) and to the long-wave read (0.80) without settling either; the derivations do (sections 2.4 and 2.5). They cannot tell the share's forms apart; that is left to the derivation of the stay (section 5). Of the eleven ends of the logarithms tested, two break a theorem (section 2.5): they are set by the exact cost of a length, and the nearest rival, at $1.0305\,\hbar c/\lambda$, is one of them; that cost weighs a displacement by no law of comparison. Apart from the end at the cell, the other eight each break a reading or a measurement and miss light by more than four errors; the couplings put all of their weight on the end at the cell. Before those theorems the rival end at $1.0305\,\hbar c/\lambda$ held 0.23 of the weight.

When the hadronic input is counted as a choice and the read is left free, a rival fits as well. On the electron–positron input 0.02760(10), the carrier's own stiffness or its mean, with the

share's form $(2K + 1)/3$, lands all three couplings within one error; on the widest menu 28 combinations fit as well as the reported one, every one of them on that input. Neither part of it survives what is now derived: the own stiffness and the mean are not reads light can take (section 2.4), and the form $(2K + 1)/3$ leaves the time bond undressed, which the lattice's symmetry forbids (section 5). With those exclusions in place the couplings put between 0.84 and 0.90 of the weight on the lattice input, and 0.99 once the ends of the logarithms that break a theorem are removed as well.

These chances take the coarse agreement as given: that light's and colour's couplings at the cell lie within one per cent of whole counts times one stiffness near 1.04 at all. If both couplings at the cell could have been anywhere between 1 and 64 on a logarithmic scale, that alone happens about once in 1,200 with the carrier's own reads at the cell, and about once in 20 with every choice of the ten-choice menu left free. Taking both levels together, the chance that some combination of that menu lands all three couplings within one error is about one in a million (1.1×10^{-6}).

The menus count the alternatives named here. While the dressing was being sought, 120 other values for it were tried: ratios of the ledger's fixed counts, laws of its draw counts, and a few closed forms; ten of them lie between 1.035 and 1.047. Added as further reads to the ten-choice menu with the coefficient and the bonds held, they raise its chance from 0.0009 to 0.005, and no combination using them fits as well as the reported one. Earlier forms for where the logarithms end, beyond the eleven ends counted here, are not included.

The package was fixed before an input fault in the carry's starting state, found for another reason, was corrected, and the correction moved light toward the measurement with nothing chosen again.

5 Colour, the weak share and the quark masses

Colour. With the same K, colour at the cell is $4K = 4.161584$ against 4.163585 ± 0.004975: -0.40. Equivalently, the strong coupling at the Z that makes colour exactly $4K$ at the cell is $\alpha_s(M_Z) = 0.11836 \pm 0.00003$ in the adopted convention (minimal subtraction at the cell would move it by about 0.001), against the measured 0.1180 ± 0.0009 [15]. Colour's window for K is loose (± 0.0012); it confirms that light and colour read one common dressing more than it tests the dressing's value.

The weak share. Light's stiffness splits into the weak part and the hypercharge part, and the split comes from a second count on the record, distinct from the rule that gives light $8K$ and colour $4K$. It reads a link's stiffness as the sum, over every carrier that moves with the link, of that carrier's index, the sum of the squared charges of its states. Matter's three generations enter at a weight w: each generation adds 2 to colour, 2 to the weak link, $10/3$ to hypercharge and $16/3$ to light. The links' own carriers enter at weight one: colour's add 3, the weak link's 2, hypercharge's none, and the charged weak carriers add 2 to light. The scalar doublet enters at a weight w_h, adding $\frac{1}{2}$ to the weak link and to hypercharge and 1 to light. So colour counts $6w + 3$, the weak link $6w + 2 + w_h/2$, hypercharge $10w + w_h/2$, and light $16w + 2 + w_h$, the weak and hypercharge counts together. This count fixes ratios only; light's $8K$ sets the size. Light is twice colour, as the loop law requires at every dressing, exactly when $4w + w_h = 4$.

Matter is read at its moving share, $w = 1 - \sigma$, where the stay σ is the share of a body's moves that rest; the line then gives $w_h = 4\sigma$, which we call D. The weak count is $8 - D$ and light's is $18 - 3D$, so

$$\frac{1/g^2}{1/e^2} = \frac{8 - D}{18 - 3D},$$

which is $7/15$ when nothing is dressed ($\sigma = \frac{1}{4}$).

The stay is read from the carrier. A recognition has sixteen moves, one for each pair of an account debited and an account credited: four are rests and twelve are transfers. A rest closes one of light's triangles (its two postings and the time bond); a transfer closes none [18]. The lattice's symmetries carry the time bond onto a posting bond, so the rest's triangle is dressed by the carrier's mean loop weight K_m. Light's turn along an open path can be removed by relabelling the phases at its events, so the carrier has nothing on a transfer to dress. If weights act on odds and a move reads the carrier only through the light it closes, the stay is $K_m/(K_m+3)$ and $D = 4K_m/(K_m + 3)$, which gives the share 0.4674803 against 0.4674982 ± 0.0000583 asked: -0.31. The reading that weights act on odds was adopted with the share in view. Had a move instead weighed the carrier's own cost on the two links it crosses, the share would move by less than a twentieth of its error. The other stay forms on record each break one of these steps: a quarter of the rest's weight does not act on odds, and a triangle with its time bond undressed is ruled out by the symmetry. Writing $D - 1 = c\,(K_m - 1)$, the form gives $c = 3/(K_m + 3)$, three quarters when nothing is dressed, because twelve of the sixteen moves are transfers. The measured share allows c from 0.70 to 0.81, so on its own it would also admit $4/5$; no count of the moves gives $4/5$.

Every step from the indices to the stay is a reading, not a derivation, and the form was adopted with the measured share in view. The share hardly depends on the dressing (its slope is 0.02 per unit of K_m), so its landing tests this form, not the value of K. The weak coupling and hypercharge at the cell are light and the share recombined, not new equations.

The rule that gives light $8K$ and colour $4K$, $1/g^2 = 2STK$ for a link read on carriers of index T, does not give the weak share. Applied to the weak link and to hypercharge separately, each read on the scalar doublet, it gives $4K$ each and a share of one half; the measured 0.4675 excludes that by about 560 errors. The two counts make different kinds of statement. The rule gives the size of one link's own law on one representation. The occupation count gives only how stiffness divides among the links. They speak to the same ratio twice. For colour against light they agree exactly when $4w + w_h = 4$, which is how the scalar's weight was fixed, so that agreement is built in. For the weak link against light they agree only at $D = 2$, far from the measured D near 1.03. The package takes light's and colour's sizes from the rule and the division of light between the weak link and hypercharge from the occupation count. Within a single cost of the two columns' link the division is fixed by its form. Such a cost is $a\ \mathrm{tr}\,X^2 + b\,(\mathrm{tr}\,X)^2$ for the link's generator X; light's stiffness is then $a + b$ and the weak share $a/(2(a + b))$. The rule's one law on each direction of the turn has $b = 0$, and so the share one half. The measured share asks for a separate cost on the link's overall turn worth 6.5 per cent of light's. Why the totals follow one rule and their division the other is an open problem.

Because the share hardly depends on the dressing, it asks for the hadronic input by itself: 0.027797 ± 0.000061, within 1.3 combined errors of the value light asks. The two errors share part of the carry, so this is a consistency, not two independent data: two of the package's

equations take the same side of the disagreement between the hadronic determinations, and one of them reads no dressing. The share's own ask lies 0.31 errors below the 2026 lattice value and 1.7 above the data-driven 0.02760(10).

The quark masses. On the ledger's mass table (work not yet published), three times each quark's number at the cell, counted in golden steps above the electron, sits on a whole place plus a small term, and that factor of three depends on colour's stiffness at the cell. Run at the long-wave value, the six quark rows give a factor of 2.9970 ± 0.0047, 0.63 of its error below three. The dressing at which the factor is exactly three, found by running the table, is 1.040244 ± 0.000241 (figure 1); the long-wave value lies +0.63 from it, the carrier's own stiffness +2.1 and the mean +2.5. The six terms of that table were fitted to the quark masses, so this is a check on fitted terms, not on fitted light; it is the one other measured equation that asks for the value of K, and the computed K lies between the two asks.

Two numbers it was never compared with. The same package predicts two quantities that played no part in choosing anything in it. With the fine-structure constant, the Fermi constant and the top and Higgs masses as measured, and the hadronic input, the strong coupling and the Z mass solved so that light, colour and the weak share hold at the cell, it gives the W mass as 80.3512 ± 0.0048 GeV and the leptonic effective weak mixing angle as $\sin^2\theta_{\text{eff}} = 0.23160 \pm 0.00005$. Both were computed, with the rule for scoring them written down, before they were compared. The 2026 averages are 80.3625 ± 0.0077 GeV, 1.25 combined errors above, and 0.23150 ± 0.00013, 0.68 below [16]; both pass. The Standard Model alone, on the same inputs, sits at 1.37 and 0.85 errors, so these two numbers test the package less sharply than the weak share does. They do test that the dressing enters the stay: with the rest undressed, the angle would miss by 4.0 errors. Against the earlier W-mass combination that includes the CDF measurement [37], the package, like the Standard Model, misses by 3.4. The Z mass the package solves for, with the Fermi constant setting the scale, is 91.1940 GeV. The measured value is 91.1876 ± 0.0021 GeV, so the package lies 6.4 MeV above it; with the carry's own 5.0 MeV error, that is 1.2 combined errors. This is the 1.3-error gap between the hadronic inputs the weak share and light ask for, seen in the Z mass. It is not an independent test.

6 Where it does not close

The closure's error is the carry's, and the largest part of the carry's error is the hadronic input $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$: how much the quark loops screen light between rest and the Z. It cannot be computed in perturbation theory at low energy. It is determined two ways, from the rate at which electrons and positrons annihilate into hadrons and from lattice computation, and the two disagree. The disagreement is the same one that divides the predictions of the muon's magnetic moment: the 2025 consensus evaluation of that moment set the electron–positron data aside for its hadronic part until the conflicts among the data sets are understood [17]. Table 2 lists the determinations and what the closure gives on each.

On the determinations that rest on a lattice computation at the physical pion mass the closure holds within 1.2 errors on four of the five, one of which lies above the value it asks; the fifth, which takes its low-energy part from the lattice data of 2017, lies 2.2 below.

determination	$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$	light	α^{-1}
lattice, Conigli et al. 2026 [38]	0.027821(49)	−1.16	137.028(7)
lattice (Mainz 2022), Erler et al. 2024 [42]	0.02794(9)	+0.64	137.044(13)
lattice (BMW), Erler and Ferro-Hernández 2023 [41]	0.027842(63)	−0.60	137.031(9)
lattice, Mainz 2022 [39]	0.02773(15)	−1.00	137.015(21)
lattice (BMW 2017), de Blas et al. 2022 [40]	0.02766(10)	−2.18	137.006(14)
world average, 2024 review [15]	0.02783(6)	−0.82	137.029(9)
e^+e^- and 2023 pions, Di Luzio et al. 2024 [43]	0.02775(12)	−1.08	137.018(17)
e^+e^- and 2023 pions, Narison 2025 [45]	0.027707(45)	−3.57	137.012(7)
e^+e^- and 2023 pions, Narison 2023 [44]	0.027663(45)	−4.47	137.006(7)
e^+e^- data to 2019 below 2 GeV, Erler et al. 2024 [42]	0.02762(5)	−4.89	137.000(7)
e^+e^- data to 2019, Davier et al., input of the 2026 fit [16]	0.02760(10)	−2.77	136.997(14)
e^+e^-, Erler and Ferro-Hernández 2017, in the review's table [16]	0.02761(10)	−2.67	136.999(14)
e^+e^-, de Trocóniz and Ynduráin, in the review's table [16]	0.02754(10)	−3.35	136.989(14)
lattice $\Pi(0)$ at $m_\pi = 185$ MeV with perturbation theory, Dominguez and Hernández 2023, five flavours [50]	0.027486(73)	−5.24	136.982(10)
electroweak fit, input left free [16]	0.02734(33)	−1.64	136.962(45)

Table 2. Determinations of the hadronic input, light's pull at the cell on each (in standard errors, the carry's and the dressing's together), and the fine-structure constant the closure gives with each.

These determinations differ as much in how they carry the lattice data to the Z as in the data themselves: the Mainz data of 2022 give 0.02773 carried with the Adler function from $Q^2 = 5\,\mathrm{GeV}^2$, and 0.02794 when only the light quarks are taken from the lattice, at $Q^2 = 4\,\mathrm{GeV}^2$, and the charm and bottom quarks and all the running above are taken from perturbation theory (an earlier calculation of the same kind gave 0.02795(11), +0.63). On the 2024 world average and on one electron–positron evaluation with the 2023 pion data the closure holds as well. The other electron–positron determinations with errors of 0.0001 or less lie between 2.7 and 4.9 errors below, and a determination that takes from a lattice computation only the light quarks' correlator at zero momentum, at a pion mass of 185 MeV, and everything else from perturbation theory lies 5.2 below. A fuller census of thirty-three determinations and fit values, from 2001 to 2026, shows the same pattern.

The weak share, which does not involve the dressing, says the same in the adopted convention: it lands at −0.31 on the lattice input and at +1.68 on the data-driven input of the 2026 fit. The two rows share the convention, the scale and the carry, so this is one convention's verdict, not two independent ones; in minimal subtraction the same form of the share would side the other way. The read does not rescue the closure on the electron–positron side: the reads that would land there are not light's (section 2.4). The closure's verdict is the lattice's against the data's.

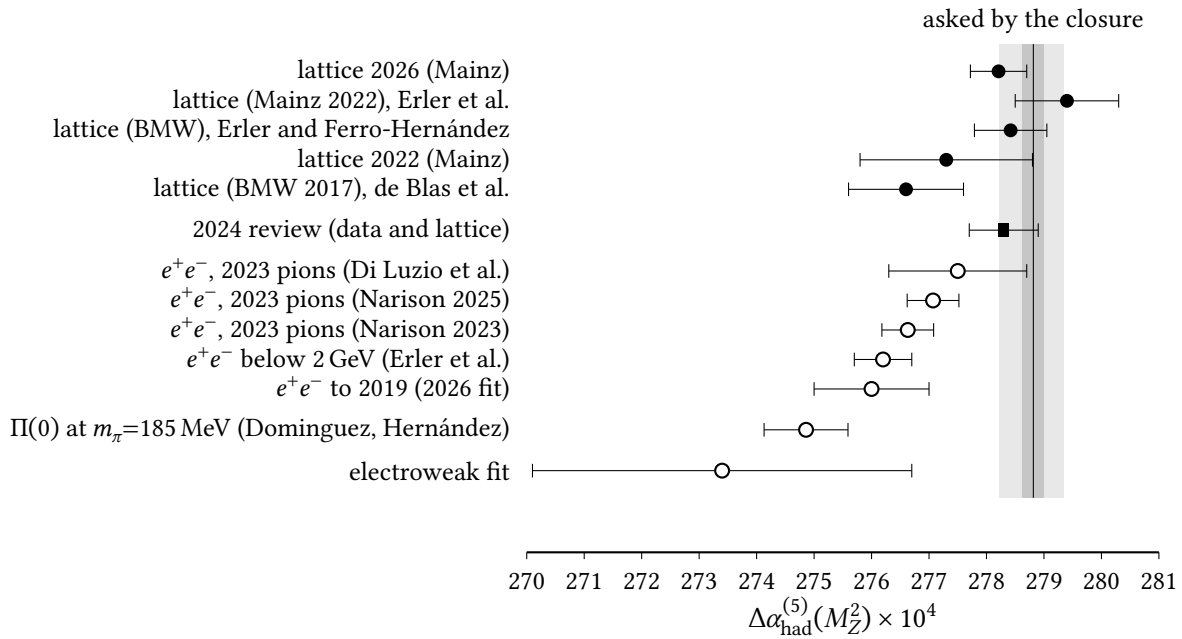


Figure 2. The hadronic input as determined (one standard error) against the value the closure asks: the dark band is the value filed on the corrected carry, 0.027882 ± 0.000019; the light band is the value filed with version 1.0, 0.027878 ± 0.000056. Filled circles: lattice computation at the physical pion mass; square: the 2024 world average; open circles: electron–positron data, a lattice value at a heavier pion mass with perturbation theory, and the electroweak fit. The filled circles lie within 2.2 combined errors of the value asked, one of them above it; the electron–positron values with errors of 0.0001 or less lie 2.7 to 4.9 below.

7 The test

The hadronic input, as filed with version 1.0. With α measured, the closure fixes the hadronic input it needs, the value that makes light land exactly. Version 1.0 filed it, on 14 November 2026, as

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027878 \pm 0.000056$$

(0.0278785 ± 0.0000561 before rounding), the error being the carry's other errors over the slope 10.888, with this rule: a new determination with an error of 0.00010 or less that lies within two combined errors of it passes; one that lies beyond three fails it, and with it the closure as stated, unless the carry itself is shown to be wrong; between two and three, it strains it. By that rule the closure fails on a new determination of error 0.00010 below 0.027534, or of error 0.00005 below 0.027653. (Computed from the printed 0.027878 ± 0.000056, the second boundary is 0.0276528, so a value between it and 0.027653 strains rather than fails.) That filing stands with its numbers.

The hadronic input on the corrected carry. The same rule on the corrected carry, with the dressing of section 2.3, is a second filing, also dated 14 November 2026:

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027882 \pm 0.000019,$$

with the measured strong coupling and top mass, or 0.027872 ± 0.000008 with the ledger's own. By it the closure fails on a new determination of error 0.00010 below 0.0275761, or of

error 0.00005 below 0.0277209 (with the ledger's inputs, below 0.0275706 and 0.0277196). Both filings score only determinations first made public after 14 November 2026: those published before were in view when the rule was written, and they lie on both sides of the value asked (section 6). The test is of the package as stated, with the long-wave read. Changing the read after a determination would be a new hypothesis, not a rescue, and the read is not free (section 2.4).

The strong coupling. The same package gives the strong coupling at the Z as 0.11836 ± 0.00003 in its convention, thirty times sharper than its present measurement. The rule for it was filed on the carry of version 1.0, where the package gives 0.11835: a new world average or determination $y \pm \sigma_y$ (minimal subtraction, five flavours, $\sigma_y \le 0.0005$) is scored by $t = |y - 0.11835| / \sqrt{\sigma_y^2 + 0.00003^2 + 0.0003^2}$, where 0.0003 is an allowance, fixed with the rule, for the carry's truncation and its matching at 200 GeV in this number: $t \le 2$ passes, $t > 3$ fails. For $\sigma_y = 0.0005$ the package fails below 0.1165984 or above 0.1201016.

The W mass and the weak angle. A later world average of the W mass with an error of 0.005 GeV fails the package as stated if it lies below 80.3303 or above 80.3721 GeV, and a later average of the effective angle with an error of 0.0001 if it lies below 0.231255 or above 0.231937. The Standard Model's own values on the same inputs sit close to the package's, so such a failure would first be the Standard Model's. Neither replaces the hadronic test.

A second measured equation: the muon. The quark loops that screen light between rest and the Z also make the largest hadronic contribution to the muon's anomalous magnetic moment, $a_\mu^{\mathrm{HVP,LO}}$. Both are integrals of the same function, the vacuum polarization of the colour-charged quarks, weighted differently: the muon reads 93 per cent of it below $Q^2 = 0.25\ \mathrm{GeV}^2$, the running of α reads it up to the Z. The closure's asked input therefore implies a value of $a_\mu^{\mathrm{HVP,LO}}$, and the measured muon tests it. The rule was fixed before computing: the comparison is with what the measured anomaly asks of $a_\mu^{\mathrm{HVP,LO}}$ once every other contribution of the 2025 Standard Model evaluation is subtracted, and a pull beyond two standard errors strains the closure, beyond three fails it.

The measured anomaly, $a_\mu = 116\,592\,071.5(14.5) \times 10^{-11}$ [46], less the other contributions of the 2025 evaluation [47], asks $a_\mu^{\mathrm{HVP,LO}} = (717.05 \pm 1.76) \times 10^{-10}$. To turn the asked input into a value for the muon we need the shape of the vacuum polarization below $Q_0^2 = 9\ \mathrm{GeV}^2$. Above it the running is perturbative (0.019304 at the lattice authors' stated inputs, plus 0.000045 from the space-like to the time-like Z), so the asked input fixes the part below it: 0.008533, which is 0.72 per cent above the lattice value 0.008472. We take the shape from the same lattice group, which also gives $a_\mu^{\mathrm{HVP,LO}} = 724.5(7.1) \times 10^{-10}$ [49], and scale the muon's value by the same 0.72 per cent. The two lattice quantities are strongly correlated (a correlation of 0.8 has been published between a lattice muon value and its running at the Z [42]), so their ratio is sharper than the muon's value alone: its error is 0.71 per cent against 0.99. The closure then implies

$$a_\mu^{\mathrm{HVP,LO}} = (729.7 \pm 6.2) \times 10^{-10},$$

1.97 standard errors above what the measured muon asks: inside the rule, at its edge. With the correlation set to one the error is 5.5 and the pull 2.21, a strain. With the ledger's own

strong coupling and quark masses in the carry the asked input is 0.027872 and the pull 1.77 (2.00 at full correlation). On the value filed with version 1.0 the pull is 1.57. Against the 2025 lattice average, $713.2(6.1) \times 10^{-10}$ [47], it is 1.90.

The lattice's shape matters as much as the number. Read the same way, every asked input between 0.02759 and 0.02788 would pass, so the muon does not by itself choose between the determinations of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$. What separates them is the shape: the 2020 evaluation of $a_\mu^{\rm HVP,LO}$ from electron–positron data, $693.1(4.0) \times 10^{-10}$ [48], lies 5.5 standard errors below what the measured muon asks, and the lattice evaluations lie within about one.

The closure asks 0.000061 more of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ than the lattice determination of 2026, and the muon reads only the part of that excess that sits below 9 GeV^2. With full correlation the muon passes if no more than 0.000047 of it sits there; at least 0.000014, nearly a quarter of it, must then sit in the perturbative running above 9 GeV^2, where the muon does not look. If the whole excess sat there, the muon's pull would be +1.2 (+1.3 with full correlation): what remains is the lattice's own muon value, which lies above the measured muon's ask.

8 The chain, step by step

The table lists every step from the ledger to light's stiffness at the cell, and the carry from there to rest, with its status in one word. *Derived*: follows from the steps above it and the premises named, and is conditional on those premises where they are readings. *Reading*: an identification adopted and built on, not yet derived; what would derive it is named. *Borrowed*: standard physics, used as it stands.

step	status	what it rests on, or what would derive it
the cell's edge $\lambda = 2\sqrt{\ln 2}\,\ell_P$	reading	one bit of record per λ^2 of area, fixed on gravity, not on light
the lattice of events, 24 bonds	reading	the ledger's postings, four accounts with two columns; given it, the 12 bonds per event and the triangles are counted
the cost $J = \cosh\ell - 1$	derived	uniqueness of the reciprocal cost [20]
the cost acts as a weight (coefficient one)	reading	not a continuous setting; light locates it at 0.9979 ± 0.0018; derived if the record's weights are shown to be its costs
each bond's fluctuation, $\langle \ell \sinh \ell \rangle = 1/12$	derived	the identity on the weight (section 2.3)
light reads the relaxed long-wave value, not the mean or the carrier's own stiffness	derived	what a coupling is: the coefficient in light's law at long waves, its links obeying their law in the carrier's weights; that energy splits into the relaxed cost and a part that does not depend on the loop value
light reads the carrier without weighing it	reading	the same reading as "the links' own turns are not summed as weights at the cell"; summed with the carrier, light's turns would move its coefficient to about 35/24 and put colour ten errors low

step	status	what it rests on, or what would derive it
the triangle weighs its links' mean	reading	each link reads a loop's whole turn (parallel); given it, the lattice's symmetry and the balance value force the mean; shared in series the harmonic mean, 22 errors low
$K = 1.040396 \pm 0.000002$	derived	computed; an earlier computation gives 1.040399 ± 0.000005
$1/g^2 = 2STK$ on the 24 bonds	derived	the lattice's triangles and their second moment
the ratio of light's count to colour's, two	derived	the ratio of their generator traces, given that light turns one of a body's two columns and colour the three partners; the rule gives light's and colour's totals only
the pair reading: a comparison is costed on its pair	reading	chosen with the couplings in view; one formula, $J(z^2)$, sets $S = 4$ and the matching's unit and leaves K unchanged; measured $S = 4.0003 \pm 0.0003$ and one entry 0.510 ± 0.011 against $\frac{1}{2}$; the open step is a calibration: two records compared, one at balance, cost the comparison cost of the other
a record holds each link's unseen part	reading	otherwise no free electron or quark; the quarks' thirds need it in any form
no matching constant	derived	the summed count (section 2.5)
the links' own turns are not summed as weights at the cell	reading	chosen with the couplings in view; a lawful weight sees no turn; summed, the turns would add constants to the matching that measurement excludes, and would make light weigh the carrier (colour ten errors low)
the unrecorded count is summed	reading	one end for every spin, 1.7 errors from zero; a cut on each line of a loop lies 41 away; derived from "nothing outside the record" on weighted histories
gravity's strain adds nothing at the cell	reading	room -0.00007 to $+0.00022$; derived from the strain's own law
the carry from the cell to rest	borrowed	the Standard Model's running, three loops with computed shifts; the ledger has the two ends, not yet the passage
no other charged body below the cell	reading	the mass table's bodies; tested by every search for new charged or coloured states
the time bond is dressed like a posting bond, so a rest weighs K_m	derived	the lattice's symmetries
a move that closes no loop keeps weight one	derived	light's relabelling freedom, given that a move reads the carrier only through the light it closes (a reading that moves the share by 0.04 of its error)
a weight acts on odds	reading	adopted with the share in view; derived if the record's move law is shown to normalize over the sixteen moves
the stay $K_m/(K_m + 3)$, $c = 12/16$	derived	from the rows above, and conditional on them

step	status	what it rests on, or what would derive it
the weak share's count: each carrier's index, matter at its moving share, the scalar's weight by the line	reading	adopted with the measured share in view; the rule $2STK$ applied to the weak link and hypercharge separately gives a share of one half, 560 errors off: why the totals follow one rule and their division another is open

A step that is a reading is called one wherever it appears in this paper. The readings were identified with the couplings in view; section 4.1 counts what that costs. Read on one side, the same structure gives $2K$ for light and K for colour, and summing the links' own turns at the cell would give the matching the constants that measurement excludes: the counts and the matching are readings, not consequences of the structure alone.

9 What it means, and what is still owed

The fine-structure constant is light's turn stiffness, read at rest. At the ledger's smallest cell that stiffness is a whole count times one computed number, and the number comes from the ledger's own carrier: twelve bonds to an event, the cost of a comparison at coefficient one, and the stiffness a long wave reads from it. Carried down to rest, it is $1/137.028$. The magic number is a count and a stiffness.

Several steps of the chain follow from stated premises: the ratio of light's count to colour's, which stiffness light reads, the absence of a constant in the matching once the unrecorded count is summed, and the dressing of a rest. Each is conditional on its premises, and the premises that carry the physics are readings. What remains to derive is short and exact: that the ledger costs a comparison on its pair, which sets both the counts and the matching's unit; that a record holds each link's unseen part; that the count of an unrecorded history is summed; that the links' own turns are not summed as weights at the cell; that gravity's strain is never summed; that the cost acts as a weight at coefficient one; that each link reads the whole of a loop's turn; and the readings that give the weak share's form, a weight acting on odds among them. Why light's and colour's totals follow the count rule while the division of light between the weak link and hypercharge follows another count is an open problem. Light's landing can be read back as the number of senses (4.0003 ± 0.0003), one entry's cost (0.510 ± 0.011 against $\frac{1}{2}$), the weight's coefficient (0.9979 ± 0.0018) or the postings to a unit (7.84 ± 0.16); these are one datum re-expressed with the other readings held fixed, not four confirmations. Colour's own landing puts the senses at 4.002 ± 0.005. The carry from the cell down to rest is the Standard Model's computed running, borrowed whole: the ledger has the two ends, and the passage between them is still the borrowed calculation. Its error is the closure's error, and its hadronic part decides the test filed in section 7.

Code and data

The computation of the carrier's three stiffnesses (the mean loop weight, its own stiffness, and light's long-wave stiffness at all orders and at second order), the stored runs, their analysis and the figure of their dependence on the lattice size are public at

`https://recognitionphysics.org/publications/fine-structure-constant/FineStructureFromTheLedger-code-v1.1.zip`

and in the paper's Zenodo record, doi:10.5281/zenodo.24235248. They are written in Python with NumPy; each run checks the lattice, and each run that solves for light its solver, against the definitions before it starts, and the archive lists the command for every run and the checksum of every file.

About this version

Version 1.1 (14 November 2026) replaces version 1.0 of the same day. The carry's error budget was recomputed with signs: one line of version 1.0's budget, the revision of the reference state, had been added in quadrature over two couplings whose moves cancel in light, so light's error is 0.000572, not 0.000811, and three computed shifts are applied. The dressing was computed a second time, independently, and the headline uses that computation. Together these move the result from 137.0281 ± 0.0102 to 137.0277 ± 0.0072; on this budget the reported package lands light, colour and the weak share within two errors, not one. The census of the hadronic input was completed: two lattice-based determinations lie above the value the closure asks, 0.02794(9) and 0.02795(11), and the sharpest electron–positron evaluations lie more than three errors below it. Colour's count is read on the pair as light's is, and the factor two between them is the trace of the generator. The second-order value 1.040378 is the one with a Gaussian covariance of the weights. New in this version: which stiffness light reads, from what a coupling is, and the readings under the weighting and the matching; the weak share's count and the open problem it leaves; the count of the trials, with the chance conditioned on the coarse structure; two numbers filed before comparison and the solved Z mass; the muon equation; a second filing of the hadronic input; and the table of the chain's steps, each marked derived, reading or borrowed. The filing of version 1.0 stands with its numbers.

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